REVISTA ELECTRÓNICA DE INVESTIGACIÓN EN EDUCACIÓN EN CIENCIAS
ISSN 1850-6666

Número especial - 20 Años REIEC

Guy Brousseau's contribution to the constitution of the didactics of mathematics[1]

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Abstract

This theoretical text analyzes the contribution of Guy Brousseau to the constitution of mathematics teaching. It presents a brief review of his academic and professional career and analyzes some interpretations of the theoretical constructs developed by this researcher within the theory of the situation. It reveals inexhaustible and constant energy, unwavering determination, boundless curiosity and extreme rigor. All this led him to develop the most successful and coherent theory of the last thirty years. The strength and uniqueness of this thought and approach were synthesized in what he termed situation theory, fueled by several constructs that attest to its scientific relevance.

Keywords: Mathematics Didactics, Situation, Situation Theory, Didactic Contract, Environment

La contribución de Guy Brousseau a la constitución de la didáctica de las matemáticas

Resumen

Este texto teórico analiza la contribución de Guy Brousseau a la constitución de la enseñanza de las matemáticas. Presenta una breve reseña de su trayectoria académica y profesional y analiza algunas interpretaciones de los constructos teóricos desarrollados por este investigador dentro de la teoría de la situación. Revela una energía inagotable y constante, una determinación inquebrantable, una curiosidad ilimitada y un rigor extremo. Todo ello le llevó a desarrollar la teoría más exitosa y coherente de los últimos treinta años. La fuerza y singularidad de este pensamiento y enfoque se sintetizaron en lo que él denominó teoría de la situación, alimentada por varios constructos que atestiguan su relevancia científica.

Palabras clave: Didáctica de las matemáticas, Situación, Teoría de la situación, Contrato didáctico, Entorno.

La contribution de Guy Brousseau à la constitution de la didactique des mathématiques

Résumé

Ce texte théorique analyse la contribution de Guy Brousseau au développement de l'enseignement des mathématiques. Il présente un bref aperçu de son parcours académique et professionnel et examine certaines interprétations des constructions théoriques qu'il a élaborées dans le cadre de la théorie de la situation. Il révèle une énergie inépuisable et constante, une détermination sans faille, une curiosité insatiable et une rigueur extrême. Toutes ces qualités l'ont conduit à développer la théorie la plus aboutie et la plus cohérente des trente dernières années. La force et l'originalité de cette pensée et de cette approche ont été synthétisées dans ce qu'il a appelé la théorie de la situation, étayée par plusieurs constructions qui attestent de sa pertinence scientifique.

[1] This article is a translation of the chapter " A contribuição de Guy Brousseau na constituição da didática da matemática ", published in the book " Teoria das situações: Fundamentos teóricos e metodológicos para a investigação e a formação profissional", published by CRV (which authorized the publication of its English in this journal.

Mots-clés : Didactique des mathématiques, Situation, Théorie de la situation, Contrat didactique, Environnement

A contribuição de Guy Brousseau para o estabelecimento da didática da matemática

Resumo

Este texto teórico analisa a contribuição de Guy Brousseau para o desenvolvimento do ensino da matemática. Apresenta uma breve visão geral de sua carreira acadêmica e profissional e examina algumas interpretações dos constructos teóricos que ele desenvolveu dentro da teoria da situação. Revela uma energia inesgotável e constante, uma determinação inabalável, uma curiosidade ilimitada e um rigor extremo. Todas essas qualidades o levaram a desenvolver a teoria mais bem-sucedida e coerente dos últimos trinta anos. A força e a singularidade desse pensamento e abordagem foram sintetizadas no que ele chamou de teoria da situação, apoiada por diversos constructos que atestam sua relevância científica.

Palavras-chave: Didática da matemática, Situação, Teoria da situação, Contrato didático, Ambiente.

1. INTRODUCTION

Research into didactics of mathematics emerged in the 1970s due to the need to understand why students failed to appropriate mathematical concepts and the inefficiency of mathematics reform. Didactics of mathematics as an area of knowledge was born by the efforts of French researchers, among others, to investigate problems related to mathematics teaching and learning and propose well-founded actions to solve, at least partially, such problems. The theoretical and experimental constructions allowed the identification and understanding of phenomena that impact the teaching and learning processes of mathematical concepts. As one of the pioneers in such research, Guy Brousseau played an important role in the construction of the didactics of mathematics theoretical models that allow characterizing knowledge and knowing, as well as their evolution, both from a historical perspective and from the identification of interactions that must be established between teacher, student, and knowing.

The objective of this text is to concisely retrace Guy Brousseau's contribution to the constitution of the didactics of mathematics. The author's project was

> *[...] deconstructing the didactics of mathematics as an experimental epistemology of mathematics, not considering the student or the teacher as the central object but the situation that organizes and conditions the interactions between students, teacher, and knowledge, a radically new project that might have seemed disproportionately ambitious when it emerged in the early sixties (Artigue, 2024, p.157, our translation)*

Artigue further asserts that

> *[...] the principles that underlie this project –the recognized need to develop a systemic approach to teaching and learning processes, to perceive mathematics learning as a combination of adaptation and acculturation processes, to develop research methodologies that take into account the complexity of teaching systems– may seem widely shared today, but it was truly visionary (Artigue, 2024, p.157, free translation)*

To explore Guy Brousseau's contribution to the constitution of the didactics of mathematics, we present a brief account of his academic and professional trajectory. Furthermore, we offer an understanding of some theoretical constructs he developed within the situation theory. We highlight that some constructs are conceptualized in footnotes due to the space reserved for this text.

This contribution is theoretical and based on works by Guy Brousseau and other sources from the mathematics teaching area.

2. GUY BROUSSEAU'S BRIEF BIOGRAPHY

Guy Brousseau, born on February 4, 1933, in Taza (French protectorate in Morocco), died on February 15, 2024, in Cambes (Gironde - France). He was one of the founders of the French branch of the didactics of mathematics.

2.1 The didactics of mathematics

It is the science of the specific conditions for disseminating mathematical knowledge necessary for people's occupations (broad sense). It deals (in a restricted sense) with the conditions in which an institution called "teaching" attempts (mandated if necessary by another institution) to modify the knowledge of another called "taught" when the latter is not capable of doing so autonomously and does not feel unavoidably the need to do so.

A didactic project is a social project that aims to help a subject or an institution acquire knowledge that has been constituted or is in the process of being constituted. Teaching comprises all actions that seek to carry out this didactic project.

Over time, the need to characterize the specificities of this didactic project in the mathematics field and more specific situations grew, culminating in the didactics of mathematics. From this need, Brousseau created the theory of didactic situations (TSD) within the didactics of mathematics. This allows for a detailed analysis of the relationships between the research situations proposed to students (and adapted to the level in question), the analysis of their productions, and their understanding of the mathematical concepts that are objects of learning (Bloch, 2024).

According to Brousseau (2000), the didactics of mathematics encompasses the activity of teaching, the art and knowledge necessary for this, the art of preparing and producing resources for this activity, the study of this teaching, and everything that is manifested in it as a social project, socio-historical fact, or as a phenomenon.

Currently, the didactics of mathematics is constituted as a science, a field of research interested in explaining the didactic phenomena that occur in teaching and learning processes, proposing techniques or models for analysis and investigation. According to Brousseau (2000), the insertion of didactics in culture allows for the political management of knowledge dissemination and thus makes its use and creation more democratic.

From this perspective, Figure 1 presents mathematics didactics as the main lens of an instrument for capturing images, which registers the images resulting from the didactic acts of the actors in the "classroom" scenario –the focus of Brousseau's study and investigation. This capture is recorded and reflected through an accordion box (represented by juxtaposed rectangles) that interconnects with the back compartment (Figure 1), which we call the research field.

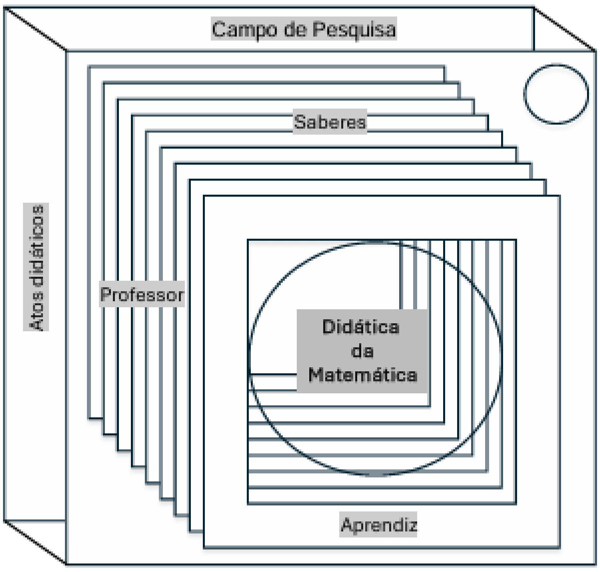

Figure 1: Didactics of mathematics.

Source: Authors

The research field is where researchers stand with their magnifying glasses. We highlight Brousseau, a researcher who has a microscopic look at the teaching situations in which the teacher, the learner, and the various relationships between these subjects, knowing, and *milieu*[2] within didactic situations.

From this perspective, we briefly outline Brousseau's academic career, scientific contribution, participation in collective activities and international exchanges, and, finally, the various dimensions of his influence.

2.2. An extraordinary career

Guy Brousseau began his academic career as a student at a primary teacher education school. He taught primary school for a few years until he was assigned to join the most diverse teams, which, at the beginning of the 1960s, embarked on the general effort to renew the teaching of mathematics.

With his administration's support, he completed his university education before being recruited as an assistant at the University of Bordeaux I. At that university, as part of the IREM[3] and with the constant support of Professor Jean Colmez, Brousseau conducted most of his research on mathematics teaching in compulsory education. He defended his doctoral thesis in 1986.

With the collaboration of academic authorities, the researcher created the COREM[4], which he directed from 1973 to 1998 before founding LADIST[5], the complementary laboratory of COREM.

The very close links between his personal work and teacher education within the IREM framework, and then the specificity and originality of his research project, led him to publish locally (18 Cahiers de l'IREM de Bordeaux, de 1969 a 1978) texts that were essential for understanding the development of the fundamental theoretical instrument represented by the theory of didactical situations. These texts, together with others published in periodicals such as

[2] We discuss some understandings of the concept of *milieu* further up.

[3] Institut de Recherche sur l'Enseignement des Mathématiques (Research Institutes on Mathematics Teaching), where prospective and in-service teachers could carry out reflection, research, and action together, combining their respective fields of specialization. Their actions were successful. The Bordeaux IREM was opportunely founded, one of the first in France, and Brousseau joined it to develop it according to his plans.

[4] Centre pour l'Observation et la Recherche sur l'Enseignement des Mathématiques (archives vidéo du COREM disponibles sur http://visa.ens-lyon.fr/visa, archives « papier » disponibles au *Centro de Recursos de Didáctica de las Matemáticas Guy Brousseau* http://www.imac.uji.es/CRDM/).

[5] *Laboratoire Aquitain de Didactique des Sciences et Technique.*

RDM, can be found in a collection published in English by Kluwer in 1997, *Theory of Didactical Situations in Mathematics*.

Guy Brousseau's passion for mathematics teaching stems from a double fascination: on the one hand, a fascination with mathematics, its explanatory power and its ability to shape thought, and, on the other, a fascination with the transmission and dissemination of knowledge and the study of the conditions that make this possible.

Throughout his scientific career, the researcher managed to unite this double passion: an inexhaustible and constant energy, unshakable determination, a boundless curiosity, and an extreme rigor that led him to develop the most successful and coherent theory in the last thirty years. The strength and uniqueness of this thought and approach emerged in the second half of the 1960s.

Brousseau made an original and decisive theoretical choice, presented in a seminal text, "Processus de mathematização" [Mathematization Processus], in a lecture at the *Journées de l'APMEP*[6] in 1970. This text is an important contribution to the construction of the TDS and the area of didactics of mathematics. Its timeliness and relevance will never be denied.

If the student and the teacher are the main actors in teaching and learning, one must also, and first of all, be interested in a third instance, a "silent actor": the situation in which they evolve, in which the student's activity and that of the teacher develop according to their respective projects: learning and teaching.

Brousseau believes a *situation* is, on the one hand, a hypothetical *game* (that can be defined mathematically) that explains a minimum system of necessary conditions under which specific (mathematical) knowledge can be manifested by the decisions of observable effects (actions) of an actant on a *milieu*. On the other hand, a model of the type above allows us to interpret the part of the observable decisions of a real subject that are related to their relationship with specific mathematical knowledge.

2.3. The emergence of the situation

One of Guy Brousseau's most significant contributions

> [...] is placing at the center of one's question not the students, the *teacher, or the elementaryization of knowledge, but the situation that unites them, and studying the conditions so that this situation guarantees the learning of knowledge by the most significant number of students. He considered that the three terms could not be dissolved and that the conditions must relate to their relationships. Furthermore, he had the ambitious project of scientifically studying these conditions based on observing the implementation of situations in the classroom, hence the development of the didactic engineering methodology and a teaching observation system in the execution process. (Perrin-Glorian, 2024, p.185)*

One *situation* as a central object of study can be characterized from two points of view. Regarding the first point of view, Brousseau (1999, s/p) gives the name situation to the "model of interaction of a subject with a "milieu" in which a specific knowledge is a resource available to the subject that allows him/her to achieve a favorable state in the "milieu"." Some situations require the construction or acquisition of necessary knowledge and schemes. Others offer the possibility for the subject to construct new knowledge independently in a genetic process. The word situation describes all situations that frame an action as a theoretical model, possibly formal, used to study it.The second point of view is based on the analysis of a non-didactic situation, i.e., the situation in which mathematics is applied either by the mathematician or by a "simple" user in a particular universe of practices.

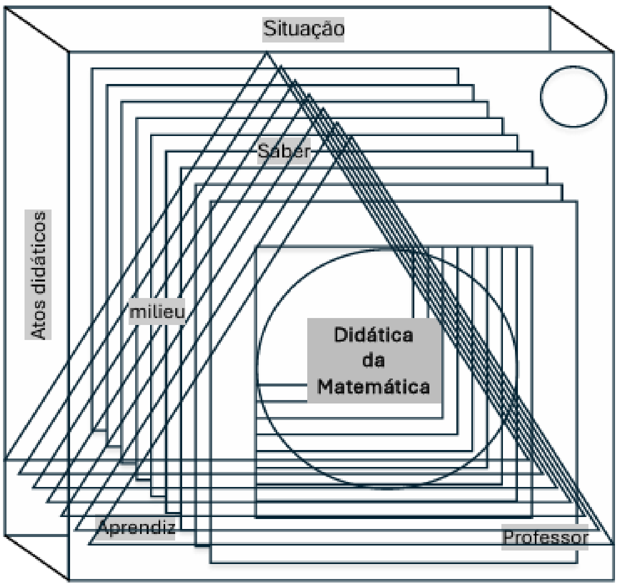

Figure 2: Situation. **Source:** Authors

6 *Association des Professeurs de Mathématiques de l'Enseignement Public*

Figure 2 presents a reinterpretation of the situation, placing it, from Brousseau's perspective, as a field of study and research. The different triangles symbolize the several interactions between the subject and the *milieu*[7], which occur in didactic and/or adidactic situations, symbolized by the compartments of the accordion box (represented by juxtaposed rectangles). We must highlight that during the teaching and learning processes of a given knowing, the knowledge acquired by the learner in a given situation propagates/projects to other situations. Therefore, Figure 2 presents the juxtaposition of the triangles to give the idea of propagation and projection of this learner's knowledge in a process that aims to institutionalize knowing at stake.

Indeed, this process that is established in knowing mathematics is not reduced to the knowledge of theorems or algorithms but rather to the recognition of the conditions in which these theorems and definitions are used. The meaning of mathematical knowledge does not depend on a set of external obligations linked, for example, to the use of specific knowledge, which is the requirement of any didactic injunction.

Based on this vision, the central theoretical perspective then consists of studying the conditions for the installation in the didactic system of situations that involve the student in the same way as non-didactic situations (adidactic situations). Its objective is to show that adidactic situations can be constructed and explain how they work theoretically and the conditions of their "didactic viability," i.e., their implementation, taking into account the restrictions of the mathematics classroom (contingency).

It is important to emphasize that a situation cannot be conceived without considering the specificity of the knowledge and its relationships with other knowledge. Elaborating situations also requires understanding the origin of the resistant difficulties observed in the learning and functioning of a class and the institutional restrictions to which it is subject (Perrin-Glorian, 2024).

2.4. Aspects of the successful implementation of Guy Brousseau's ideas

The first aspect concerns the implementation itself, which led Brousseau to propose a new concept –devolution: if knowledge precedes the student, understanding it requires a use that, although expected by the teacher, cannot be dictated to him/her. It is the paradox of devolution: "If the teacher says what he/she wants, he/she can no longer get it" (Brousseau, 1998, p. 73). Brousseau addressed this paradox in the 1960s when he studied the conditions for overcoming it by returning adidactic situations to students. What basic strategies can students develop in this situation? What feedback can they receive? What didactic variables are likely to maintain the meaning of the targeted knowledge? etc.").

The second aspect is closely linked to the first, as it concerns the conditions to maintain students' commitment to the situation. Based on a clinical case (now famous in the mathematics teaching community, the "Gaël case[8]"), Brousseau studied the set of reciprocal obligations that each partner in the didactic situation imposes or believes they impose on the others and those imposed on them or that they believe are imposed on them regarding the knowledge at stake: this is the concept of a didactic contract. It results from a "negotiation," often implicit, of how relationships are established between a student, a given *milieu*, and an educational system.

This contract is not real; it is neither explicit nor freely agreed, as it depends on knowledge that is necessarily unknown to the students. It places the teacher and students before a true paradoxical injunction: if the teacher says what he/she wants students to do, they can only obtain it by fulfilling an order and not by exercising their knowledge and judgment. On the other hand, if students accept that the teacher teaches them the solutions and answers, they do not solve them alone and, therefore, do not acquire the necessary mathematical knowledge. However, learning requires refusing the contract so that students can autonomously control the problem (devolution).

Mercier (2024, 180) notes that the didactic contract is

> *[...] the attribution of reciprocal intentions to teach and learn. [...] makes it possible to describe the strategies observed in the didactic game with two actors. This notion requires a resumption of the model: students' game is certainly modeled as a game against the milieu, but it is played in the context of a didactical situation whose effects are a function of a didactic contract. The form of the contract, specified by the transactions relating to each teaching object, therefore, determines* ***the epistemology of acquired knowledge*** *(translation and emphasis added).*

From this perspective, learning is not based on how well the contract works but on how it is broken; hence, it is vital to study the conditions under which it is broken. It is as an actant (acting actor) in the situation that the subject encounters knowledge, but this is not enough for there to be learning because, although students' experience is a necessary condition, this knowledge in action must also be identified as such, labeled, and added to socially recognized knowledge.

Guy Brousseau thus highlights the need for institutionalization and opens a new field for theorizing

7 In this text, we will use the French term "milieu" or "milieux" instead of "mean" as we understand that this does not account for the idea at stake. In this chapter, we will explain what we understand by "milieu" according to Brousseau (1986).

8 "Gaël is an intelligent child who fails in mathematics. It was one of nine cases studied between 1980 and 1985 (at COREM in Bordeaux). Observing him in the classroom and proposing several didactic and didactic situations, the author raised the hypothesis that Gaël used a strategy to avoid the "conflict of knowing"... which he calls "hysteroid type avoidance", while other children showed "obsessive avoidance" (these behaviors should not be confused with the psychiatric categories of the same name, which are serious personality disorders). It was possible to propose psychological explanations for this behavior, but they did not provide any means of correcting the avoidances and these explanations centered on the researchers' interest in studying a characteristic of the child or his abilities, rather than remaining at the level of the behaviors and conditions that caused them or that could change them. These behaviors reflect the child's refusal, conscious or not, to accept their share of responsibility in the act of making decisions in a teaching situation, and therefore learning, in front of an adult." (Brousseau, 1999 s/n, in: https://guy-brousseau.com/1201/le-cas-de-gael-2009/ , free translation)

teaching phenomena, whose experimental ordeal is the object of study in the next section.

2.5. The theory put to the test of facts in COREM

One of Guy Brousseau's main concerns is to carry out an experimental study of the phenomena of mathematics teaching, a scientific project that is part of a general scheme based on interaction with the objects studied, with these objects being apprehended within the structure of an adapted theoretical paradigm. Here, theory cannot say what should be. It models facts, summons facts, and makes phenomena emerge to analyze and interpret them.

In an article published in 1978, "The Observation of Didactical Facts,"[9] Guy Brousseau provides a solid foundation for the method at the heart of his work. This method was built around observation applied to the field of didactics: the idea was to create collections of facts and construct them as didactical phenomena, as well as to study their reproducibility, degree of generality, and consistency.

COREM, whose principal Guy Brousseau had defined in the late 1960s, was created with the support of public authorities from 1972 onwards, which allowed him to carry out this study. This research structure, which, unfortunately, remained unique in France, functioned until the end of the 1990s.

COREM is the result of a combination of a primary school with a structure that can accommodate research and observation of classroom situations proposed by researchers. These situations were designed and constructed based on the didactical situation theory, specific questions and hypotheses derived from the research, and the experience of teachers responsible for the classes. The theoretical and practical notion of didactic engineering reflects the functioning of a system based on close collaboration between teachers and researchers. Furthermore, and supporting this scientific project, Guy Brousseau contributed to the development of the use of statistics in research on mathematics teaching both from a heuristic perspective (multidimensional analyses, for example), as well as in the creation and use of implicative analysis (Régis Gras, 1979) and hierarchical classification similarity in teaching (Lermann, 1981), in addition to testing theoretical hypotheses (inferential statistics, descriptive statistics, and data mining methods).

2.6. The mathematical fields studied from a didactical point of view

Whether directly, through his or his students' works or through work carried out within the study paradigm he identified, Guy Brousseau was interested in all areas of mathematics, especially those encompassing compulsory schooling: the difficulties of learning the classic multiplication and division algorithms, the meaning of the operation and the construction of the algorithm, the first lessons on numbers and numeration, the *fundamental situation*[10] of numbers as a means of creating a collection equipotent with a given collection, which, combined with the use of didactic variables, allows the generation of many dominated situations of *action or communication*[11], making it possible to structure initial learning successfully. In addition to these themes, the following stand out:

a) *Odds* at the end of elementary school: encountering *situations where the first notions of probabilities are decision-making tools* (G. Brousseau).

b) Rational numbers and decimal numbers: fundamental situations and a complete annual progression based on a multi-year program (G. Brousseau & N. Brousseau, 1987).

c) The necessary diversity of contexts and situations in which mathematical reasoning becomes specific: solving school arithmetic problems, multiple choice situations, etc. (P. Gibel, 2004; P. Orus, 1992; B. Mopondi, 1992).

d) Determining the place of non-formalized prior knowledge and its effective use in teaching: the case of geometry (R. Berthelot & M.-H. Salin, 1992; D. Fregona, 1995), enumeration (Joel Briand, 1993) and reasoning (P. Orus, 1992).

e) The teaching of subtraction and the family of situations based on the box game (G. Brousseau).

f) The study of transition conditions between school arithmetic and algebra (D. Broin).

g) The notion of function and the role of graphic representation (Pedro Alson, Haran, 2000; Isabelle Bloch, 2000; E. Lacasta, 1995).

h) The beginnings of proportionality: a fundamental situation based on fair sharing (E. Comin, 2000).

Guy Brousseau's commitment to teaching mathematics, monitoring it, and studying the questions it raises was not limited to research. At a national level, he played an extremely important role, mainly in the Association of Mathematics Teachers, through which he actively participated in the conception and creation of IREM. These are original institutions in the French institutional context, from which several collaborations were developed in the service of mathematics teaching, based on three poles: research, innovation, and teacher education.

Brousseau was directly behind the creation of COPIRELEM[12], a national working group that has brought

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[9] Observation of didactic facts.

[10] Understandings of the "fundamental situation" construct will be addressed in the section reserved for the study of theoretical constructs developed by Guy Brousseau.

[11] Both constructs will also be in the section indicated above.

[12] Commission Permanente des IREM sur l'Enseignement Élémentaire. COPIRELEM, Commission Permanente des IREM sur l'Enseignement Élémentaire, is made up of around twenty members from different academies. They are responsible for educating teachers in mathematics and didactics of mathematics (initial and in-service education) and are involved in research on the didactics of mathematics. Since its creation in 1973, COPIRELEM has had a twofold mission: a) On the one hand, to bring together and centralize the work of the various IREM elementary groups on the teaching of mathematics in primary schools and on pre-service and in-service education in mathematics for teachers of primary schools; COPIRELEM, Commission Permanente des IREM sur l'Enseignement Élémentaire, is made up of around twenty members from different academies (see list of members). They are responsible for teacher education

together primary school teacher educators over the past 30 years. Furthermore, Guy Brousseau played a very active role in creating several other instruments of collective scientific action dedicated to young researchers' education and debate and circulation of ideas: among them, we must mention the scientific journal RDM[13], the professors' association ARDM[14], the Summer School and the National Seminar on Didactics of Mathematics.

Guy Brousseau also became involved internationally, following in the footsteps of Caleb Gattegno, Jean Piaget, Willy Servais, Zofia Krygowska, Lucienne Félix, and Hans Freudenthal. His actions were fundamental in creating the CIEAEM[15], of which he was secretary for several years and regularly accompanied during his "summer trips" from Switzerland to Mexico and from Hungary to Great Britain from 1960 to the early 1990s. One can observe the diversity and depth of his work in a structure as free from institutional restrictions as possible, such as CIEAEM during the 1960s, 1970s, and 1980s.

Guy Brousseau played a central role in launching the international PME[16] group at the ICME[17] in Karlsruhe in 1976. He was regularly invited to participate in collective work and international scientific events related to mathematics teaching.

2.7. Tools for teaching action, teacher education, and research

Guy Brousseau's influence extends far beyond the limits of research. Teacher education has always been one of Guy Brousseau's concerns. In the 1970s, for example, within the scope of the INRP and the IREM, several teams were created to develop experimental teaching products and generalize them through books for teachers and student manuals. These products were largely based, on the one hand, on the theoretical structure provided by the theory of didactical situations and, on the other hand, on the countless proposals for situations and problems constructed and studied in COREM.

The recognition of the role and place of students' mathematical activity as the driving force of learning, consideration of epistemological and didactical obstacles[18], the organized support for fundamental situations, and attention to formulations are achievements that strongly influence the curricula and practices of French teachers.

Brousseau's concepts, tested in terms of their ability to promote intelligent didactic action, have had a strong influence on current elementary school teacher education programs in France and other countries**.** This influence can also be observed in teacher recruitment.

Students who wish to become teachers can analyze students' works and pedagogical documents, relying on analytical categories derived from the theory of didactical situations.

This influence can also be observed in other phases of education, during which young teachers learn other aspects of their profession: the construction of teaching and learning situations. His contribution to the creation of the COPIRELEM (whose work he followed from the beginning), ensured that primary school mathematics had a unique instrument for the national coordination of teacher education, linked to the IREM and the IUFM[19].

2.8. Guy Brousseau – Felix Klein Medal 2003

In June 1997, Guy Brousseau received the title of Doctor Honoris Causa from the University of Montreal for his significant contribution to mathematics teaching and teacher education.

The first Felix Klein Medal[20] of the International Commission on Mathematical Instruction (ICMI) was awarded to Guy Brousseau for his essential contribution to the development of mathematics education as a field of scientific research through his theoretical and experimental work in the area for around forty years.

The award also rewards his constant efforts throughout his career to ensure that this research improved the mathematical education of students and teachers.

3. MAIN CONSTRUCTS DEVELOPED IN THE FIELD OF DIDACTICS OF MATHEMATICS

Figure 3 represents, in a non-exhaustive way, the main theoretical constructs developed by Guy Brousseau. The different understandings of these constructs are briefly presented, starting with the theory of situations.

in mathematics and didactics of mathematics (initial and in-service education) and are involved in research on the didactics of mathematics. Since its creation in 1973, COPIRELEM has had a twofold mission: On the one hand, to bring together and centralize the work of various elementary groups of IREM on mathematics teaching in primary schools and on pre-service and in-service education in mathematics for primary school teachers; secondly, to encourage research on sensitive or contingent points linked to institutional changes (curricula, school organization, initial education, etc., our translation). (in: https://www.copirelem.fr/, accessed: 04/29/2024).

13 Recherches em Didactique des Mathématiques.

14 Association pour la Recherche en Didactique des Mathématiques.

15 Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques.

16 Psychology Mathematics Education.

17 https://www.mathunion.org/icmi/icme/icme-international-congress-mathematical-education

18 Brousseau (1989) states an obstacle is knowledge that produces adequate responses, in a certain context, in each context, but can lead to errors outside of it. For example, the rule that allows comparing natural numbers does not always lead to a correct answer when comparing two rational numbers written in decimal form. The author further asserts that this knowledge resists the contradictions with which it is confronted and the establishment of new knowledge.

19 Instituts Universitaires de Formation des Maîtres.

20 https://ardm.eu/qui-sommes-nous-who-are-we-quienes-somos/guy-brousseau-medaille-felix-klein-2003/

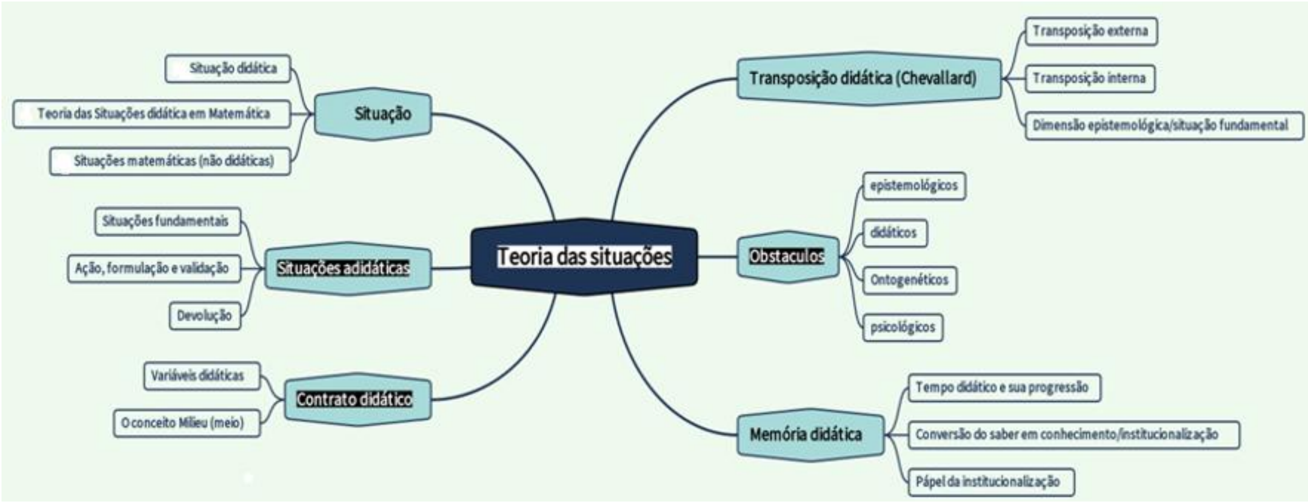

Figure 3: main theoretical constructs developed by Guy Brousseau

Source: Construction with assistance from https://gitmind.com/app/docsl

1.1 Situation Theory

One objective of the situation theory is to create a model of the interaction between the learner, knowledge, and the *milieu* in which learning should occur. Guy Brousseau developed the situation theory to model the teaching and learning processes of mathematical concepts. According to the author,

> *a learning process can be generally characterized (if not determined) by a set of identifiable (natural or didactic) reproducible situations, often leading to modifying a set of students' behaviors, a characteristic modification of the acquisition of a given knowledge. (Brousseau, 1975, apud Almouloud, 2022, p.33)*

Therefore, situation theory (ST) aims to characterize a learning process through a series of situations that can be reproduced and generally lead to modifying students' behaviors at stake in the desired situations. This modification testifies to the possible acquisition of certain knowledge, of the occurrence of significant learning.

Figure 4 presents the situation theory as a theoretical construct in the research field that, under Brousseau's lens, can analyze the different types of possible situations in the teaching and learning processes of specific knowledg

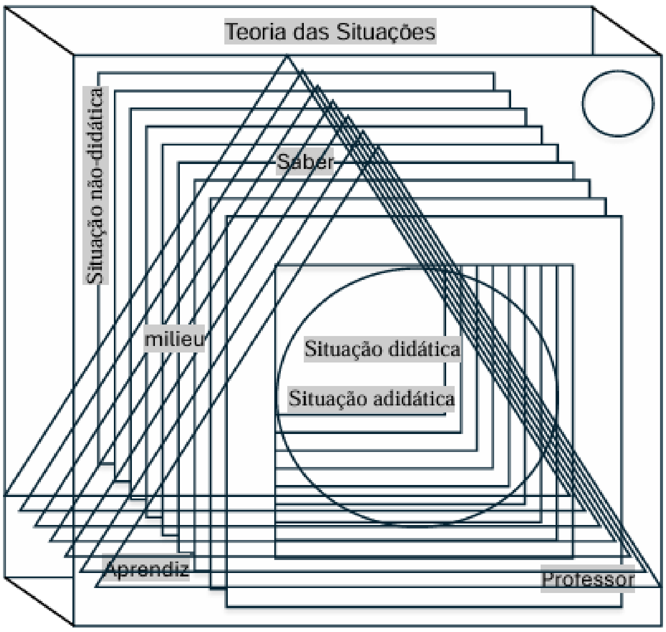

Figure 4: Situation theory. **Source:** Authors

We must highlight that the central object of the ST study is the didactical situation, in which the interactions established between teacher, student, and knowledge are identified. Brousseau (1986) seeks to theorize the phenomena linked to these interactions, aiming at the specificity of the knowledge taught. To this end, he considers the structure formed by the minimal system, which is the set of interactions between teacher and students mediated by knowledge in teaching situations, fundamental.

In situation theory, the consistency of objects and their properties (logical, mathematical, ergonomic), necessary for the logical construction and invention of "situations," is studied, and scientific (empirical or experimental) comparisons of the adaptation of these models and their characteristics with contingency are made.

Bloch (2024, p. 163) asserts that TDS

> *[...] promotes the teaching of fundamental mathematical concepts at a certain level through appropriate situations that encourage significant research phases on the students' part. The basis of didactical theories is research on knowledge, how it is taught, and how it is learned; in the TDS, this includes, after the construction of situations implemented in the classroom, qualitative analyses of the situations and the students' work. Therefore, there is a focus on mathematical topics, their difficulties, obstacles, and the learning process.*

The author also notes that the models built

> *in TDS, function as tools to answer essential questions: How do we analyze the difficulties inherent in teaching and learning a mathematical sign and concept? What is the appropriate situation for teaching this concept so that students do not just learn it by rote? What are didactic phenomena? How can they be observed and reported? And how can students understand a concept and its usefulness in solving problems? (Bloch, 2024, p.163)*

Brousseau emphasizes the importance of understanding the distinctions between didactic, adidactic, and non-didactic "situations" (Figure 4). A non-didactic situation specific to knowledge is a situation without a didactic purpose, in which the relationship with knowledge is developed as an economic means of action; therefore, in Figure 3, this type of situation is not located within the didactic triangles. An adidactic situation (Chart 1) has a didactic purpose (i.e., organized by the teacher) in which the subject acts as if the situation were non-didactic (the subject responds independently of the didactic purpose). Therefore, elements in the didactic situation form an adidactic *milieu* that is antagonistic to the student.

3.2. *Milieu* in adidactic situations

Brousseau (1998) states the *milieu* is the antagonistic system of the actant. About an action situation, the *milieu* is everything that acts on the student or/and on which the student acts.

The author believes the actant is "that" which acts on the *milieu* rationally and economically within the rules of the situation.

> *The structuring of students' didactic milieu reveals an interlocking of situations corresponding to distinct projects, each serving as a milieu for the next. The milieu of a mathematical concept is the aggregate of milieux of situations in which knowledge related to this concept appears as a means of resolution. Example: the sheet of paper, the graduated ruler, and the compass generate the milieu of Euclidean plane geometry. (Brousseau, 1998, s/n, Glossaire V5, free translation, emphasis added)*

For a generic student, the *milieu* can also be defined as a knowledge system capable of evolving, of learning a specific knowledge. For Perrin-Glorian (2024, p. 185), "The distinction between knowledge and knowing is closely related to the creation of an adequate *milieu* that allows the transformation of the knowing that is intended to be learned into the knowledge necessary to solve a problem that represents that knowing." She further asserts that this characterization of the *milieu* implies identifying problems that can be solved by knowing and how new knowledge can emerge when seeking to solve such a problem, relying on old knowledge and interpretations of feedback from the *milieu*.

3.3. Fundamental situation and devolution

A fundamental situation is defined as the one capable of generating the set of situations corresponding to a particular knowing through a set of didactic variables that determine it. When it can be identified, such a situation offers teaching possibilities but, above all, a representation of knowing through the problems in which it intervenes, allowing the meaning of the knowing to teach to be restored.

For Almouloud (2020), a fundamental situation is an adidactic situation whose notion for teaching is the most appropriate response. It allows knowledge to be introduced into the classroom in a properly scientific epistemology. Therefore, it is an adidactic situation characteristic of knowing or knowledge, whose functioning depends on the values of the chosen didactic variables.

The adidactic character of a fundamental situation also depends on the teacher returning to the student the learning situation, an act by which the teacher obtains that the student accepts and can accept to act in an adidactic situation (as a non-didactic model), assuming the risk and responsibility of their actions in uncertain conditions. Brousseau (1990) ensures that, in this process,

> *The teacher seeks to ensure that the student's action is produced and justified by the needs of the milieu and their knowledge, not by the interpretation of the teacher's teaching procedures. Devolution consists of the teacher not only proposing to the student a situation that should provoke an activity that has not been agreed upon but also making the student feel responsible for obtaining the proposed result and accepting the idea that the solution depends only on the exercise of their previous knowledge. The student accepts responsibility under conditions that an adult would refuse because if there is a problem and then the creation of knowledge, it is because they first have doubts and feel ignorant. This is why devolution creates responsibility but not blame in case of failure (see the paradox of deconcentration) (Brousseau, 1998, s/n, our translation)*

Devolution is the counterpart of institutionalization. These are the teacher's two didactic interventions in the "student-*milieu*-knowledge" situation. It is a critical sui generis element of the didactic contract.

.3.3. Theory of (non-didactic) mathematical situations

Brousseau (1997) asserts that situations differ essentially by the rules that are generally determined by the knowledge to teach. However, the forms of knowledge, their learning or acquisition depend on the structure of the system, i.e., of the classes of situations that can provoke learning and acquisition of knowledge through the articulation of cognitive and didactic variables[21]. From this perspective, the author presents four classes of situations (Table 1 and Figure 5): action situations, communication situations (formulation), demonstration or social validation situations, and institutionalization situations.

Manifestação	Função	Forma de conhecimento	Modo de aquisição
Ação: Performance Competência Código	Decisão Modelo implícito de ação Repertório	Meio de tomar decisões	Assimilação Acomodação
Formulação: Performance Competência Código	Oral, escrito, gestual Mensagem Linguagem	Meio de comunicar	Modificação ou criação de uma linguagem
Validação: Argumentação Prova	Enunciado Conjetura Teorema Teoria	Meio de convencer, de provar	Retórica Lógica Demonstração
Institucionalização	Saber	Referência Validação social ou cultural	Informação Convenção Instrução

Table 1: Non-didactic situations (but with didactic use) Source: Almouloud, 2022, p. 39

Figure 5 presents the didactic situation explaining how interactions occur between the subject and the *milieu* amid action, formulation, validation, and institutionalization situations. The fundamental one is represented by the thickness of the box (Figure 4), constituting the adidactic situations necessary for the acquisition of knowledge for the institutionalization of knowing.

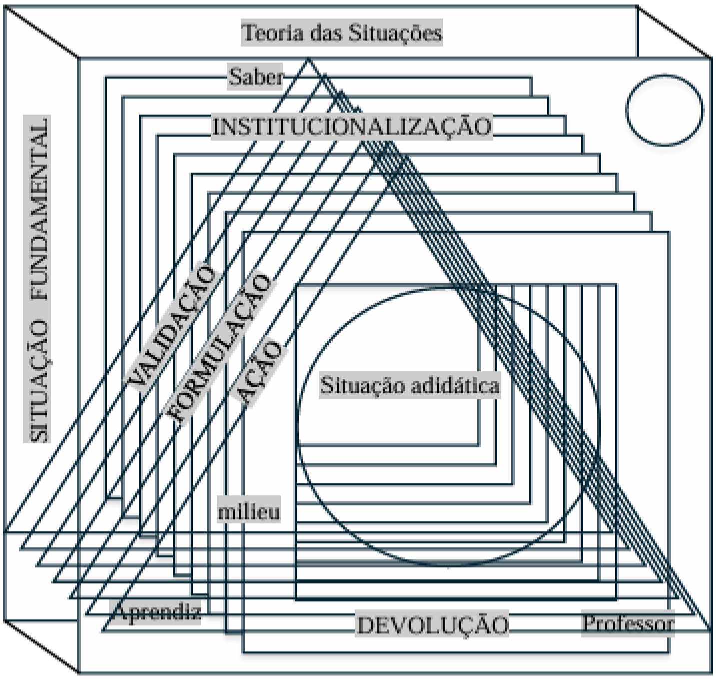

Figure 5: Situation theory. **Source:** Authors

The action situation is a "model" in which a subject manifests their knowledge in interaction with a "milieu" according to the "rules" or in the context of a situation.

In the formulation situation, knowledge formulation implements several linguistic repertoires (syntax and vocabulary), and the acquisition of these repertoires follows that of the knowledge they express, but in distinct processes. As a result, knowledge acquisition can be done directly, as

[21]**A cognitive variable is** a parameter of a situation that, according to the values that are attributed to it, alters the knowledge necessary for solving a problem and/or learning processes. A **didactic variable is a cognitive variable** that the teacher can modify and that affects the hierarchy of resolution strategies (by cost, validity, and complexity). In other words, a **didactic variable** of a problem or situation is a variable whose **values can be altered by the teacher** and whose modifications can appreciably provoke students' behavior in terms of learning, as well as provoke different procedures or types of response (Almouloud, 2016, p. 121).

in the scheme of action, or by converting into "implicit models" of acquisitions obtained through formulations and communications.

In a demonstration or social validation situation, students organize statements into demonstrations, build theories – sets of reference statements– and learn how to convince others or be persuaded without giving in to rhetorical arguments with authority, seduction, self-love, intimidation, etc.

Institutionalization is the transition from knowledge, from its role as a tool for resolving a situation of action, formulation, or proof, to a new role as an object of reference for collective or personal future uses. The teacher recognizes and names interesting knowledge in the students' productions, who must forget their own formulations and retain the vocabulary. The teacher and students enter a new convention: the value of knowledge at stake is not established immediately; it is socially and culturally guaranteed. It will reveal itself from now on in other didactic activities or not. The teacher agrees with the students that he/she may demand that they know and be familiar with certain "knowings" in the future. Learning objectives are sometimes chosen, and their responsibility is divided between the teacher and the students.

Almouloud (2022, p. 42) asserts that

> *Each situation can make the agent evolve, but it can also evolve in turn so that the genesis of knowledge would result from a (spontaneous or not) succession of new questions and answers in a process that Brousseau calls dialectics. In these processes, successions of action, formulation, and proof situations can be combined to accelerate spontaneous or voluntarily provoked learning.*

The author further states that:

> *The modeling of real teachings leads to the combination of two situations: some didactical situations present on a learning object situations that are partially freed from direct interventions –the adidactic situations. A situation models an agent's stakes and decision-making options in each "milieu." It is chosen so that the resolution strategy can only be implemented thanks to a specific mathematical knowledge. It is highly unlikely that this decision would emerge without the agent using the intended knowledge. (Almouloud, 2022, p. 38)*

The set of relationships involved in these different situations (Table 1) is a "game" related to the theory of didactical situations in mathematics (TDSM), whose objective "is to classify the situations (action, formulation, validation, and institutionalization) (Table 1) and, therefore, knowledge according to their relationships and the learning and teaching possibilities they offer" (Almouloud, 2022, p. 39).

3.4. Theory of didactical situations in mathematics (TDSM)

The TDSM is a theorization of the process of determining mathematical knowledge by a problem for which this knowledge is the solution. There are many situations related to the same knowledge. Likewise, much knowledge can be involved in a single decision.

One object of the TDSM is to classify situations and, consequently, knowledge according to their relationships and the learning and teaching possibilities they offer.

Brousseau (1997, apud Almouloud, 2022, p.43) states that the types of teaching situations –or didactic contracts– are determined by the explicit or implicit distribution of responsibilities between the teacher and the students. The teacher who wants to provoke learning situations must change their students' decision-making systems when faced with a set of situations in accordance with established knowledge. From this perspective, Brousseau (1995, 1997) identified four types of contracts, each characterized by different contracts: non-didactic contracts, didactically weak contracts relating to "new" knowing, highly didactic contracts relating to new knowing, contracts based on the transformation of ancient knowing (Figure 6).

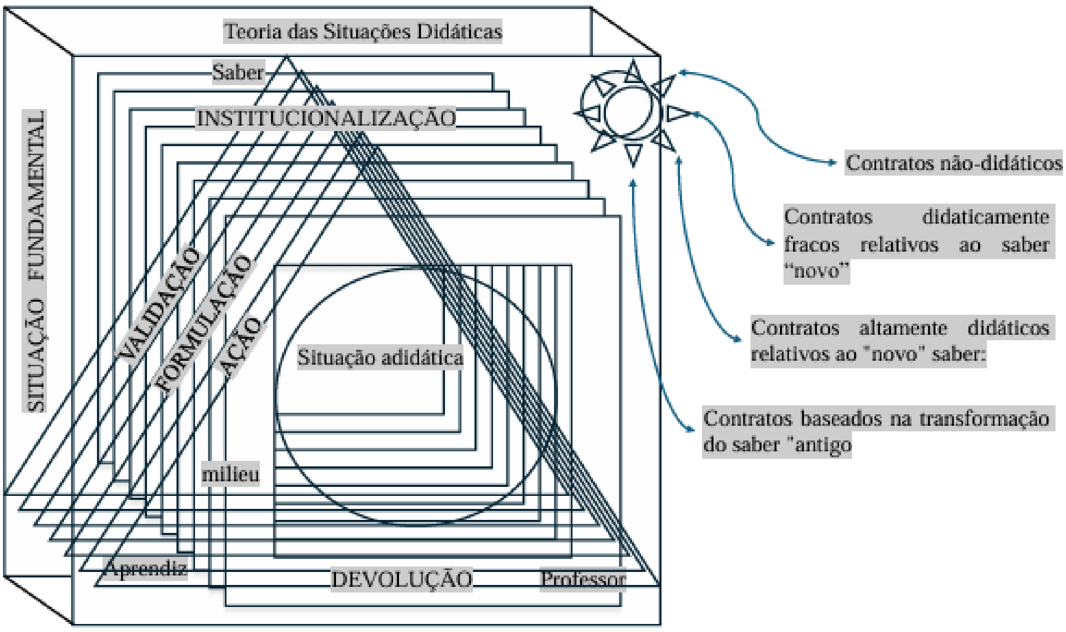

Figure 6. Theory of didactical situations in mathematics (TDSM). Source: Authors

Figure 6 is formed by the constituent elements of the situation theory and presents a reinterpretation of the role of the contract by analogizing it to the flash of an image capture instrument, which brightens the image to be recorded. The recorded image is the situation, and the flash on this situation signals the identification of four types of contracts and/or the need to break these contracts during interactions within the situations. Below is an explanation of the types of contracts:

Type 1: Non-didactic contracts consist of the transmission contract, communication contract, validity contract (expertise), and production contract. They are characterized by not considering the processes that the learner goes through to acquire knowledge/knowing or their expectations regarding the knowing at stake.

Type 2: Didactically weak contracts relating to "new" knowing comprise the following contracts: information contract, didactic contract for the use of knowledge, initiation or control contract, didactic instruction, or study direction contract. In these contracts, the student is the leading responsible for effective communication, which must be carried out per a process in which the disseminator of knowledge assumes increasing responsibility. In these types of contracts

> [...] The apprentice decides on the use of the means at their disposal. Their 'instructor' looks for the main statements of the theory, surrounded by lemmas and corollaries, application problems of various types, exploration or training exercises, and assessment means. Brousseau (1997) states that this set constitutes a fictitious but formal means of instruction the teacher makes available to the learner. This epistemological fiction is part of shared knowledge. (Brousseau, 1995, apud Almouloud, 2022, p. 51)

Figure 7 summarizes the contracts in this category and their function

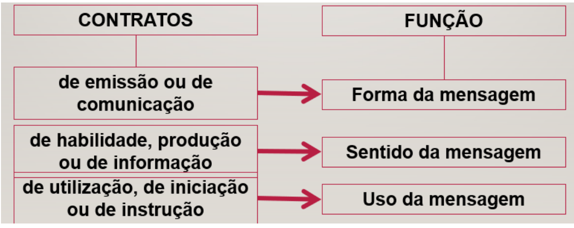

Figure 7: Not-too-didactic contracts and their functions.

Source: Almouloud, 2022, p. 52

The project of not-too-didactic contracts is to make an interlocutor appropriate knowledge, an interlocutor who is taken as an epistemic subject but not as an effective subject (Almouloud, 2022). In terms of effective didactic relationships, we often have

> *phases in which teachers' and students' responsibilities are shared according to variations of the not-too-didactic contract: emission or communication contract for the form of the message, validation, production, or information contracts for the content, application, initiation, or instruction contract for the use of the message issued. (Brousseau, 1997, apud Almouloud, 2022, p. 52)*

Type 3: Highly didactic contracts relating to "new" knowledge: The strict didactic contract, formal reproduction contract, ostension contract, conditioning contract, Socratic maieutics, empirical learning contracts, and constructivist contracts.

In the strict didactic contract, the teacher creates conditions that promote learning. For this to have any chance of happening, it modifies the learner's decision systems before "a set of typical situations, favorable to the predicted adaptation and/or according to established knowing. This change is most often made independently of the student's will and can escape their immediate control" (Almouloud, 2022, p. 53).

The manifestations of the *formal reproduction contract* lie in the fact that the student is not one of the actors in the process of building their knowledge, as the teacher does not consider the means of production of the student's responses. The teacher gives more importance to the activity that they consider the source and evidence of learning.

Brousseau (1997) asserts that in teaching focused on the *ostension contract*, the teacher sets up a scenario to visit the works (here the mathematical objects, for example), and students undertake to "visit the work," considering it representative of a class of knowings that they can use in other circumstances.

During the *conditioning contract***,** the teacher organizes the distribution of repetitive exercises and manages the flow according to the performance of the expected procedures, which are quite weak. In this type of contract, the focus is on the causes of learning without worrying about the reasons for this learning process (Almouloud, 2022).

In *Socratic maieutics*, the teacher chooses questions whose answers require mobilizing the knowledge of the student, who organizes it in a way that adds new knowledge to their cultural heritage. The teacher's initial -open or closed-questions are modified (or reformulated) depending on the students' responses.

The objective of a teaching situation focused on the *empirical learning contract* is that knowledge is supposedly learned through simple contact with the *milieu* to which the student must adapt; that is, the *milieu* is responsible for the learning process. Brousseau (1995) asserts that, in this contract, students learn what is not perceived through repetition of "direct contacts with the "milieu" that teaches, leads to learning "by" imitation, and at least "in" the context, or by "facilitation"" (Almouloud, 2022, p.57).

In the *constructivist contract*, the teacher organizes the *milieu* that involves the targeted students' knowing and learning processes and delegates responsibility for knowledge acquisition to them.

Almouloud (2022, p.58) asserts that the situation theory highlights the insufficiency of each of these contracts to build both knowing (a mathematical knowing, for example) and "knowledge that accompanies them, and practices that characterize implementation, over the course of often long geneses." In the didactic relationship, the teacher reveals himself through "the choice, rupture, and replacement of contracts according to indices and regulatory strategies currently beyond our means of investigation."

Type 4: Contracts based on the transformation of "ancient" knowledge: From the perspective of this type of contract,

> the didactic system accepts questioning the empirical order, the axiomatic order, or the standard cultural organization to adapt to a genetic order. It accepts the reality of learning through accommodation, the existence of obstacles, and the need for provisional, 'transposed' knowledge that can be reviewed in the teaching process. The articulation and genesis of collective or personal knowing enter the contract negotiation. (Brousseau, 1995, apud Almouloud, 2022, p. 58)

The author also adds that the teacher considers their own history and that of the learner, as they

> develop their own relationship with ancient knowing, to which they have already attributed a meaning, a place in the construction of other knowings. The recovery of ancient knowing calls for a new division of responsibilities between teacher and student. Most of the time, the reasons for recovery are not the same for the teacher and the students. (Almouloud, 2022, p.59)

According to Brousseau (1990), the didactic contract represents the implicit rights and duties of students and teachers concerning the objects and mathematical knowing taught. It characterizes the set of rules that students and teachers share and that limit each party's responsibilities in relation to the mathematical knowing being taught.

In teaching a knowing, the rules of communication between students and teachers about objects of knowing are established, altered, broken, and re-established as the knowing is acquired from its evolution, from its history. These rules do not take on a single, fixed form over time but result from constantly renewed negotiation. This negotiation produces a kind of game whose provisionally stable rules allow the protagonists, particularly the students, to make decisions with the confidence necessary to guarantee the characteristic independence of appropriation (Brousseau, 1986).

The antagonism of the didactic contract depends on the situations developed, especially fundamental situations.

3.5. Didactic memory

Brousseau and Centeno (1991) developed the didactic memory construct to highlight its relevance in teaching practice. Thus, the authors note that each elementary act of teaching consists of causing the student to adapt to the learning processes, that is, a lasting change in their ability to respond in different *milieux*. Learning is conceived as an *information acquisition* and a more or less profound change (accommodation or assimilation) in how this information is processed.

> *According to a classical conception, this learning is, therefore, a "memory" on the part of the student of various skills and information, which can manifest themselves in different ways (behaviors), the most important of which are identified as knowledge or knowing. (Brousseau and Centeno, 1991, p. 169, our translation, emphasis added)*

Brousseau and Centeno (1991) state that this phenomenon is particularly related to adidactic situations, as the problems that students actually encounter are the basis for teaching progress, which is not the case with teaching, whose functioning is regulated by official progress in the presentation of a text of knowing that provides summoned memorial references. This type of teaching does not require a teacher's highly developed didactic memory (Centeno, 1995).

Brousseau and Centeno (1991, p. 170) assert that the system must adapt to variations: "on the one hand, to differentiations—, on the other, to temporary evolutions, both in knowledge and institutions and in individuals (student or teacher)." Therefore,

> *[...] we can assume that, like any organization living in similar circumstances, it will have to remember specific facts related to its capabilities, to use them when making a subsequent decision. Just as in the past all memory activity was transferred to the student, one could think of concentrating all the system's memory on its main reader, the teacher. (Brousseau and Centeno, 1991, p. 170)*

The authors further state that memorization of the didactic system consists of temporary modifications of the four leading systems: transient learning for students, teachers' temporary adaptations, but also fleeting adjustments of the knowledge taught or even the *milieu* and cultural practices. One objective of reflections on teaching is to analyze the extent to which the system must adapt to the learning of a given knowledge, at a given time, for a given student, to clarify the place, in this adaptation, of specific information relating to the common past of those involved in learning.

The construction of the concept of didactic memory was based on the search for answers to the following questions:

> *Can we define and observe a teacher's didactic memory? Does it exist in a spontaneous state? Does memory play an important role in teaching? A facilitating or complicating role? Is it easier or less difficult, depending on the pedagogy? Whether it is used more or less depends on the teachers themselves, regardless of the method? Are there facts that teachers should remember? And others they should forget? Are reminders necessary at certain stages of learning? Is their use more necessary in some methods than others?* Are they more necessary for some children than others[22]? (Brousseau e Centeno, 1991, p. 170-171)

Fluckiger and Mercier (2002) point out that several research studies in didactics of mathematics show that the didactic memory evoked by a student or teacher can be used to produce an object, which is then remembered again. Now

22 To access the answers given to these questions, consult Brousseau and Centeno (1991).

integrated into the classroom, this object must have visible relevance so that students can use it again.

Brousseau and Centeno (1991, p. 2003) draw attention to the complexity and importance of:

> *Describing the nature and functioning of this memory, providing the means to observe it, distinguishing it from a body of knowledge or teachers' permanent strategies, recognizing its content-specific components, and identifying phenomena in which it plays an important role.*

The authors believe that the functioning of the didactic memory affects the understanding of mathematical questions because the effect of the system's didactic memory on the student is to give them the possibility of mobilizing knowledge that they did not adequately possess, knowledge that they could not have used on their own and that will allow them to make sense of the issue they are dealing with. The authors also note that the functioning of the didactic memory could be improved if teachers received adequate didactic knowledge to apply what students have not yet learned but have experienced with them. The teacher could thus recontextualize knowledge during learning.

3.6. Didactic transposition and the situation theory

Verret (1975) introduced the concept of didactic transposition, but it was Chevallard (1991) who was the precursor of this construct in the didactics of mathematics. Without going into detail about Chevallard's study, it focuses on the perspective of the aforementioned construct in the situation developed by Brousseau. This author questions the methodology of teaching mathematics, which is based on axiomatic presentation and allows students and the teacher to organize their activities and accumulate, in a minimum amount of time, enough *knowing* closer to wise knowing. The author also notes that this presentation:

> *[...] completely hides the history of this knowings, i.e., the succession of difficulties and questions that caused the appearance of fundamental concepts, their use in creating new problems, the introduction of techniques and questions born from the progress of other sectors, the rejection of specific points of view considered false or inappropriate, and the numerous changes that this knowing has undergone. (Brousseau, 1986, apud Almouloud, 2022, p.139)*

Therefore, scientific/sociocultural knowledge must be adapted and transformed to make it teachable.

Brousseau (1986) situates the process of didactic transposition in three stages: (1) the work of mathematicians who, when communicating the results of their research, *depersonalizes*, *decontextualizes*, and "de-temporalizes" their results; (2) *the* student's work must sometimes be analogous to that of a mathematician, i.e., they must use mathematical tools (definitions, theorems, axioms, etc.) to solve problems. The student must be faced with problem situations (chosen by the teacher) that have the potential for the student to act, formulate, prove, build models, etc.; (3) the work of the teacher, who is responsible for constructing problem situations in which the mathematical knowledge at stake "is recontextualized and re-personalized with a view to becoming student's knowledge, i.e., a more natural response to particular conditions, indispensable conditions for this knowledge to have a meaning" (Almouloud, 2022, p.139).

Brousseau (1986) gives three conditions to achieve that objective:

> -*Simulate a mathematical "microsociety" in the classroom to provoke a scientific debate, master formulation and validation situations; institutionalize the cultural and communicable knowing that we want to teach students; students must also decontextualize and recontextualize their knowing. (Almouloud, 2022, p.139)*

In the situation theory paradigm, the concept of didactic transposition is specified and operationalized by the notion of the fundamental situation of knowledge. This notion constitutes a privileged instrument for the study of transpositive phenomena, specifying the conditions for maintaining the meaning of knowledge during its transposition (ARDM, s/d, s/n).

4. CONCLUSION

Brousseau's academic and professional career is marked by his passion for mathematics, teaching mathematics, and studying and researching the conditions for disseminating this knowledge. This passion culminated in relevant contributions to mathematics teaching, such as the development of critical theoretical constructs.

This text briefly explains Brousseau's theoretical constructs based on an analogy to an image capture instrument, where Brousseau's sensitive and investigative look focused on the situation. This focus narrowed the field of vision to the explicit or implicit relationships between the learner, the teacher, and the I.

We can say that the narrowing is the result of experimentation with teaching situations experienced (captured) by Brousseau, which highlighted different types of situations: didactic, non-didactic, and adidactic situations, which culminated in the definition of the fundamental situation and constituent elements of the situations, such as the concept of *milieu*, devolution, and teaching contract. These constructs are key elements of the model for analyzing situations arising from teaching and learning processes and constitute the situation theory.

The development of Brousseau's situation theory imprinted theoretical references to the didactics of mathematics to weave reflections on the teaching and learning of mathematical concepts.

Resuming some of the reflections made during an interview given to Rezende and Moran (2022) about the model in force in the 1970s, I would say that the ST (Brousseau, 1986) created the first rupture with the dominant model of the time when considering mathematics the essence of didactic phenomena. The second rupture concerns the construction of a science whose objective is to study the factors related to mathematics teaching and learning. The third rupture is epistemological, related to the classical view of knowing, as it assumes that mathematical knowledge can only be understood through the fulfillment of an activity whose resolution appeals to this knowledge.

According to Brousseau (1986), mathematics is, above all, an activity that takes place in a situation and against a *milieu*, an activity in which different situations can be highlighted: action, formulation, and validation, as well as devolution and institutionalization (Almouloud, 2007). Furthermore, in the ST, the social and historical dimensions are fundamental aspects of knowledge acquisition. From this perspective, Brousseau (1986) postulates that knowledge acquisition must result from adapting the subjects to the situations that the teacher has organized, in which interactions with other students will play an important role.

As seen in this text, we present several theoretical constructs, such as didactic situation, adidactic situation, fundamental situation, didactic contract, and devolution, which were fundamental in the construction of the ST, and constitute theoretical tools to support the researcher in the analysis of teaching and learning processes of mathematical (and of other areas) concepts.

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