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Institutional and epistemological conditions for a transition to the paradigm of questioning the world[1]

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Abstract

This article examines the institutional and epistemological conditions for a transition from the paradigm of visiting works to the paradigm of questioning the world within educational institutions, particularly in secondary and tertiary education. Anchored in the Anthropological Theory of the Didactic, the analysis conceives paradigms as tacit social contracts that govern human activity and investigates how didactic systems evolve through changes in the relations between teachers, students, and knowledge. The discussion traces the historical development of successive variants of the first paradigm—centered on the study of pre-established works—toward the emergence of the second paradigm, characterized by collective inquiry into generating questions. Central to this transition are the notions of adidacticity, milieu, and definalization, which enable a shift from prescribed exercises to the genuine study of questions. Through examples such as study and research activities and study and research paths, the paper analyzes the dialectic of didactization and adidactization, the construction of the milieu, and the evolving topogenesis of teachers and students. The article concludes by identifying epistemological attitudes required of teachers and learners to engage in education as a process of questioning the world—problematizing, Herbartian, procognitive, exoteric, and encyclopedic—thereby redefining both the aims and the assessment of education in this emerging didactic paradigm.

Keywords: Adidacticity; Definalization; Milieu; Study and research paths; The Anthropological Theory of the Didactic.

Condiciones institucionales y epistemológicas para una transición hacia el paradigma del cuestionamiento del mundo

Resumen

Este artículo examina las condiciones institucionales y epistemológicas para una transición del paradigma de la visita de las obras al paradigma del cuestionamiento del mundo dentro de las instituciones educativas, particularmente en la educación secundaria y terciaria. Anclado en la Teoría Antropológica de lo Didáctico, el análisis concibe los paradigmas como contratos sociales tácitos que rigen la actividad humana e investiga cómo los sistemas didácticos evolucionan a través de los cambios en las relaciones entre docentes, estudiantes y saberes. La discusión traza el desarrollo histórico de las variantes sucesivas del primer paradigma—centrado en el estudio de obras preestablecidas—hacia la emergencia del segundo paradigma, caracterizado por la indagación colectiva orientada hacia la generación de preguntas. En el centro de esta transición se sitúan las nociones de adidacticidad, medio y desfinalización, que posibilitan el paso de los ejercicios prescritos al estudio genuino de cuestiones. A través de ejemplos como las actividades y los recorridos de estudio e investigación, el artículo analiza la dialéctica de la didactización y la adidactización, la construcción del medio y la topogénesis evolutiva de docentes y estudiantes. El texto concluye identificando las actitudes epistemológicas necesarias en docentes y aprendices para comprometerse con la educación como un proceso de cuestionamiento del mundo—problematizador, herbartiano, procognitivo, exotérico y enciclopédico—redefiniendo así tanto los fines como la evaluación de la educación en este paradigma didáctico emergente.

[1] This article is an expanded version of the chapter titled "Condições de uma transição para o paradigma do questionamento do mundo" that appeared in the 2022 anthology *Percursos de estudo e pesquisa à luz da teoria antropológica do didático: Fundamentos teórico-metodológicos para a formação* (Vol. 1, pp. 27–58), edited by S. A. Almouloud, R. B. Guerra, L. M. S. Farias, A. Henriques, J. M. V. Nunes and published by CRV.

Palabras clave: Adidacticidad; Desfinalización; Medio; Recorridos de estudio e investigación; Teoría Antropológica de lo Didáctico.

Conditions institutionnelles et épistémologiques pour une transition vers le paradigme du questionnement du monde

Résumé

Cet article examine les conditions institutionnelles et épistémologiques d'une transition du paradigme de la visite des œuvres au paradigme du questionnement du monde au sein des institutions éducatives, en particulier dans l'enseignement secondaire et supérieur. Ancré dans la Théorie Anthropologique du Didactique, l'analyse conçoit les paradigmes comme des contrats sociaux tacites qui régissent l'activité humaine et étudie l'évolution des systèmes didactiques à travers les transformations des relations entre enseignants, élèves et savoirs. La discussion retrace le développement historique des variantes successives du premier paradigme—centré sur l'étude d'œuvres préétablies—vers l'émergence du second paradigme, caractérisé par une enquête collective orientée vers la génération de questions. Au cœur de cette transition se trouvent les notions d'adidacticité, de milieu et de définalisation, qui rendent possible le passage d'exercices prescrits à l'étude véritable de questions. À travers des exemples tels que les activités et les parcours d'étude et de recherche, l'article analyse la dialectique de la didactisation et de l'adidactisation, la construction du milieu et la topogenèse évolutive des enseignants et des élèves. Le texte conclut en identifiant les attitudes épistémologiques nécessaires aux enseignants et aux apprenants pour s'engager dans l'éducation comme un processus de questionnement du monde—problématisant, herbartien, procognitif, exotérique et encyclopédique—redéfinissant ainsi à la fois les finalités et l'évaluation de l'éducation dans ce paradigme didactique émergent.

Mots clés: Adidacticité ; Définalisation ; Milieu ; Parcours d'étude et de recherche ; Théorie Anthropologique du Didactique

Condições institucionais e epistemológicas para uma transição rumo ao paradigma do questionamento do mundo.

Resumo

Este artigo examina as condições institucionais e epistemológicas para uma transição do paradigma da visita das obras para o paradigma do questionamento do mundo dentro das instituições educacionais, particularmente no ensino secundário e superior. Fundamentada na Teoría Antropológica do Didático, a análise concebe os paradigmas como contratos sociais tácitos que governam a atividade humana e investiga como os sistemas didáticos evoluem por meio de mudanças nas relações entre professores, alunos e conhecimento. A discussão traça o desenvolvimento histórico de variantes sucessivas do primeiro paradigma — centrado no estudo de obras preestabelecidas — em direção à emergência do segundo paradigma, caracterizado pela investigação coletiva orientada para a geração de perguntas. No cerne dessa transição estão as noções de adidaticidade, meio e desfinalização, que possibilitam a mudança de exercícios prescritos para o estudo genuíno de questões. Por meio de exemplos como atividades e percursos de estudo e pesquisa, este artigo analisa a dialética entre didatização e adidaticização, a construção do ambiente de aprendizagem e a topogênese evolutiva de professores e alunos. O texto conclui identificando as atitudes epistemológicas necessárias para que professores e alunos se engajem com a educação como um processo de questionamento do mundo — problematização, herbartiana, procognitiva, exotérmica e enciclopédica — redefinindo, assim, tanto os objetivos quanto a avaliação da educação dentro desse paradigma didático emergente.

Palavras-chave: Adidacticidade; Desfinalização; Ambiente de Aprendizagem; Percurso de Estudo e Pesquisa; Teoria Antropológica do didático.

1. INTRODUCTION

Human activity within educational institutions develops under epistemological, institutional, and cultural conditions that shape what can be taught and learned, and how. Over time, these conditions give rise to relatively stable forms of organization that define what teaching and studying mean within a given epoch. Yet as knowledge and society evolve, these forms may become insufficient for sustaining a genuine encounter between students and the world. This tension raises the question of how educational institutions might undergo a transition from one didactic paradigm to another. In the Anthropological Theory of the Didactic (ATD), a *work* is any entity created by humans—material or symbolic, practical or theoretical. Works embody earlier human responses to questions posed to the world, and they constitute the main objects around which study and teaching have traditionally been organized. Studying, in this sense, amounts to visiting works.

From this perspective, the present article addresses the question of how a transition might occur from what can be called *the paradigm of visiting works*—in which teaching and learning are centered on the study of pre-established works—to the *paradigm of questioning the world*, which foregrounds collective inquiry into generating questions. Anchored in the ATD, the analysis explores how the evolution of didactic systems depends on transformations in the relationships between teachers, students, and knowledge, and on the redefinition of the institutional contract that binds them.

The generating question of our study can therefore be stated as follows:

Under what conditions is a transition from the paradigm of visiting works to the paradigm of questioning the world possible, locally and globally?

Let us stress that what we describe in this study concerns secondary and tertiary education, as well as each of us in our research activities. But we believe that an adaptation to primary education would be possible.[2] The question raised is a difficult, complex question, to which today we are only able to provide elements of answers, in a largely conjectural way.

Following this introduction, the paper is structured as follows. Section 2 presents a concise account of the foundational concepts that underpin our theoretical perspective. Section 3 outlines the theorizing strategy adopted in the inquiry, specifying the analytic steps through which the two paradigms are modeled and compared. Section 4 develops the core exposition by examining the internal structure of the paradigms of visiting works and questioning the world, reconstructing their possible variants, and analyzing how transformations in milieu, adidacticity, praxeologies, and topogenesis condition the transitions between them. Section 5 provides a discussion of the conditions under which such transitions may occur, both locally and globally, and reflects on the implications of our analysis as well as directions for further research foundational concepts.

2. FOUNDATIONAL CONCEPTS

This section offers a concise account of several foundational concepts that structure the theoretical perspective adopted in this study.

To simplify without disfiguring the notion, we will say that a paradigm is a contract governing a certain type of human activity. One can thus speak of an artistic paradigm, a scientific paradigm, a sports paradigm; and, of course, a school paradigm or a didactic paradigm. By *contract* we mean a set of clauses that define what the governed activity consists of, what its purposes and authorized means are, and, by consequence, what cannot be done without "breaking the contract." Yet these "clauses" are generally not formulated explicitly. As Jean-Jacques Rousseau noted in *On Social Contract or Principles of Political Right* (1762/1988, p. 92), "although they may never have been formally set forth," the clauses of the contract are "everywhere tacitly accepted and recognized."

The notion of *milieu* originates in the Theory of Didactic Situations (TDS; Brousseau, 2002), where it denotes an antagonist system of material and symbolic objects, constraints, and feedback mechanisms with which students interact when solving a problem, without relying on the teacher to indicate or confirm the knowledge at stake. In the ATD—the theoretical framework to which this inquiry belongs—the concept of milieu is retained but reformulated through the dialectic of media and milieus (or of conjectures and proofs). In an inquiry on a question Q, students are confronted with statements coming from various media (textbook, teacher, internet, peers, etc.). These statements are treated as conjectures: they are based on incomplete evidence and call for justification. To seek proof of a statement is, in this perspective, to question other media that, with respect to this statement, can be treated as an adidactic milieu, that is, as a system which does not itself intend to prove or disprove the statement but simply produces reactions to the actions carried out on it. The milieu thus appears as that part of the world, made present by a medium, which can be interrogated so that conjectures may be tested, confirmed, or invalidated.

As with the milieu, the notion of *adidacticity* originates in the TDS (Brousseau, 2002), where it refers to a property of the milieu: in certain phases of a situation, the milieu can regulate students' actions without manifesting any didactic intention. The adidacticity of the milieu lies in its capacity to provide objective feedback by functioning as an antagonist to students' proposals, requiring them to justify the knowledge at stake with respect to the milieu rather than the teacher. In the ATD, adidacticity remains a property of the milieu but is interpreted within a broader institutional ecology (see e.g., Chevallard, 2019). A milieu functions adidactically when, in the course of an inquiry, it can be treated as a system that does not aim to confirm or refute a statement, yet produces reactions that allow conjectures to be tested, weakened, or validated. In this sense, adidacticity identifies those moments when responsibility for advancing the inquiry is at least partially devolved to students through a milieu capable of sustaining the conjecture–proof dynamic without didactic intervention.

In the ATD, the notion of *praxeology* provides the fundamental scheme for modeling mathematical activity and the institutional forms through which such activity is organized. A praxeology represents a relatively stable form of activity by specifying a set of task types recognized within an institution and the techniques used to address them; types of tasks and techniques constitute the *praxis* block of the praxeology. Any technique requires reasons that make it acceptable or preferable, expressed through a technology that articulates and justifies the technique. This technology, in turn, relies on more general principles that form the theory. Together, technology and theory make up the *logos* block of the praxeology. In this way, a praxeology unites practice and discourse in a single theoretical unit, allowing mathematical objects to be analyzed as products of the praxeologies that generate and sustain them.

The term *topos* comes from the Greek word for "place" and, within the ATD, refers to the institutional position occupied by a participant in a didactic system, together with the responsibilities attached to that position. As articulated in Chevallard (1999), *topogenesis* designates the process through which these positions and responsibilities are distributed, negotiated, and transformed during study. It concerns "who can or must do what" at a given moment of the inquiry—who may formulate questions, propose techniques, evaluate statements, or institutionalize results. The notion of topos and topogenesis belong to the ATD, but they resonate with the TDS through the attention both

[2] Note that in France, the sixth and seventh grades are part of secondary education.

theories give to the teacher's devolution of responsibility and to the autonomy granted to students in engaging with a milieu. Topogenesis thus provides a vocabulary for analyzing how the didactic contract allocates roles and how these allocations may shift over time. In the context of paradigm change, topogenesis is crucial: moving from the paradigm of visiting works to that of questioning the world requires new distributions of responsibility, including forms of student initiative and teacher support that allow praxeologies and milieus to function differently.

In this study, we will focus on what will be called *school* paradigms. By school we mean a *place* (now possibly virtual) where, at certain *times,* people come to *study*. The questions of what is meant by "study" and what are the very objects that are being studied, are at the heart of the "big question" discussed here. Let us note that, in general, forms of study depend on the objects being studied but tend to survive them due to a certain didactic inertia of school paradigms. Conversely, the objects studied depend on the forms of study adopted and therefore, in the long run, tend to perpetuate themselves. Of course, this curricular "congealment" is also determined by factors other than the mere inertia of forms of study.

The theoretical concepts outlined here provide the background for analyzing the transformation of didactic systems across paradigms. Before addressing this analysis in Section 4, the next section presents the way our inquiry proceeds.

3. THEORIZING STRATEGY

The present study is theoretical rather than empirical and follows a systematic analytic procedure consistent with the ATD. It is characterized by an institutional analysis of didactic activity, aimed at reconstructing and comparing the institutional principles that underpin two distinct didactic paradigms—the paradigm of visiting works and the paradigm of questioning the world. The inquiry proceeds by identifying and analyzing the conditions under which didactic systems evolve from one paradigm to another. It focuses on three closely related dimensions of didactic systems: the institutional positions of teacher and students and the toposes associated with them—that is, the types of tasks each position is entitled or required to undertake; the construction and functioning of the milieu; and the dialectic of didactization and adidactization that governs their interactions. Particular attention is given to the transformations in topogenesis that accompany these processes, as the teacher's and students' positions are redefined through their engagement with the milieu.

The study draws on examples from the design of study and research activities and from historical developments in educational institutions. These examples serve not as empirical evidence in the strict sense, but as *didactic phenomena*—observable configurations that make visible the underlying structures of didactic organization. In this way, the methodological stance remains faithful to the theoretical orientation of the ATD, whose aim is to render human didactic activity intelligible by modeling the relations between institutions, persons, and knowledge. We now turn to the development of the theoretical exposition.

4. THE TRANSFORMATION BETWEEN DIDACTIC PARADIGMS

It is necessary, first of all, to clarify the meaning of the expressions "paradigm of visiting works" (Paradigm 1) and "paradigm of questioning the world" (Paradigm 2), because this will give us an early idea of the depth of the transformations involved in moving from Paradigm 1 to Paradigm 2. For the interested reader, a concise account of the predecessors of Paradigm 1 can be found in Chevallard (2015).

4.1 The paradigm of visiting works and its variants

4.1.1 Visiting works: From applications to exercises

To address the "big question" of the historical transition from Paradigm 1 to Paradigm 2, we will further simplify the givens of the problem. In the first school paradigm that we will consider, P_1^0, a variant of Paradigm 1, the objective is the study of works $\mathcal{w}$ belonging to a fixed set $\mathcal{W}$ of works. What is essential in P_1^0 is that the reason for studying the work $\mathcal{w}$ is nothing other than the work $\mathcal{w}$ itself. To put it another way, $\mathcal{w}$ is regarded as a work of great value, recognized as such by the curricular tradition, and whose study is recognized as having a high formative value. The study, here, takes the form of a "visit," as in a museum, where one is interested in the work $\mathcal{w}$ for itself and in itself, and not for what it allows one to think and to do.

P_1^0 is in a way the "primitive" variant of Paradigm 1. The P_1^+ variant achieves a first evolution towards Paradigm 2, although it firmly falls within Paradigm 1. What difference is there between P_1^0 and P_1^+? While in variant P_1^0 the works $\mathcal{w}_1$, $\mathcal{w}_2$, $\mathcal{w}_3$, ..., are considered in and for themselves, in an almost aesthetic approach, in variant P_1^+ there appear *applications* of these works, sometimes introduced in the form of *typical examples*. In other words, efforts are being made—unevenly—to specify their possible *utility*, what they can be "used for" in order to understand the world and act within it.

The variant P_1^+ is classic, but it is relatively fragile, as the applications of a work change over historical time. Here is an example. The following theorem has long been taught: If a and b are non-zero numbers, the following equality holds:

$$\frac{a}{b} = a \times \frac{1}{b}.$$

This is a work for which many mathematics teachers today would likely struggle to identify a genuinely useful application that is not merely constructed for illustrative purposes. Yet here is the reason for the interest that one could have, a century ago and more, for this equality. As a general rule, it made it possible to transform a *division*, namely a / b, an operation traditionally regarded as difficult and even "scholarly," into a *multiplication*, namely $a \times (1 / b)$. Thus to calculate $57 / 8$, one calculated 57×0.125; to calculate (approximately) $89 / 11$, one could calculate 89×0.09 (or 0.091); to divide by 9, one could multiply by 0.11 (or 0.111);

and so on.[3] This technology (in the sense of the ATD) made it possible, among other things, to divide by the number π: one then had to memorize the fact that $1 / \pi \approx 0.32$ (or 0.318), just as, not so long ago, one had to memorize the fact that $\pi \approx 3.14$ (or 3.1416). To draw on the ground, with a rope, a circle with circumference of 10 meters, you must take a radius of length $\frac{10 \text{ m}}{2\pi} = \frac{5 \text{ m}}{\pi} = 5 \text{ m} \times (1 / \pi) \approx 5 \text{ m} \times 0.32 = 1.6 \text{ m}$. Of course, today this technology is outdated. The Google calculator, for example, immediately gives the following approximation:

(5 meters) / pi =
1.59154943 meters

Because of the aging of the "applications" as illustrated by the previous example, the variant P_1^+ tends to become corrupted and revert to the "primitive" variant of Paradigm 1, namely P_1^0. It is then important to note that one way to compensate for—and mask—this "regression" is to transform the applications, now deprived of their function, into simple "exercises" whose purpose is to train students to "manipulate" the work w taught—for example the equality $\frac{a}{b} = a \times \frac{1}{b}$. The applications of w have thus become simple training exercises that are no longer "to be learned" but "to be done"—to ensure the "good mastery" of w. We thus arrive at a variant of Paradigm 1 which we denote by P_1^{+-}. The variant P_1^{+-} of Paradigm 1 has had a long stability: it has been placidly dominant until at least the 1960s. In comparison to the variant P_1^+, it is less sensitive to possible criticisms regarding the *praxeological* relevance of the "applications" studied—will they be useful to students in understanding the world and acting within it?—because only the *formative* relevance of the exercises into which they have been transformed will matter, if at all.

4.1.2 The coming to life of activities

As we will see, to explain the passage from P_1^{+-} to the variant of Paradigm 1 that we will denote below by P_1^{++}, it is necessary to take into account the *conditions and constraints* affecting the society, or rather the *civilization* in which the society under consideration is located. To begin, given a work w, we will consider here three main institutional positions: first, that of *creator* of the work, then that of *user* of it, and finally that of *visitor* to the work. In the variant P_1^0, the students are visitors to the work and even *passive* visitors, like visitors to a museum, in the following sense: they are shown the work w, they have to get to know it, to comment on it, but cannot "manipulate" it. In the variant P_1^+, the students are, momentarily at least, supposed users of the work, which they manipulate, if only slightly, within the framework of the work's "applications." In the variant P_1^{+-}, the students cease to be putative users of the work: they become visitors again, but "active" visitors, through the manipulations they carry out in the exercises associated with the work.

During the 1960s, in many countries, the variant P_1^{+-} was subverted by socio-economic changes and their cultural repercussions: students must be emancipated from an archaic didactic structure reflecting the old social order where "subjects" submit to "masters." While the class traditionally began with the teacher's lesson and was followed by exercises for the students, in the new variant, P_1^{++}, it now begins with an *activity* meant to introduce the students, both didactically and praxeologically, to the work w.

If we look at the variant P_1^{++} as an ancestor of Paradigm 2, it is however disappointing. Often, in fact, the activity that is supposed to introduce students to w has little to do with w and appears as a simple warm-up in preparation for the teacher's subsequent presentation on w. The change, which promotes a variant that we will denote by P_1^{+++}, imposes to put forward the study of a question q of a certain type Q, to which the answer $\boldsymbol{a}$ elaborated by the class under determined conditions and constraints relies essentially on the use of the work w —which will show that w "serves" to answer questions of type Q.

4.2 Milieu and adidacticity: A conceptual revolution

The variant P_1^{+++} of Paradigm 1 has been particularly studied by Guy Brousseau in his *theory of didactic situations* in mathematics (TDS; Brousseau, 2002). An important question in this framework is the following: when students are actively studying a question q, what do they nourish their study activity with? For this, they use their cognitive equipment and their praxeological equipment as these have been constituted until then. But they also use a set of resources composed of diverse works, both material and immaterial, which constitute what will be called the *milieu* of the study M.[4] Is the teacher y of the class $c = [X, y]$ (where X is the set of students) an element of the milieu M? Here we touch on the thorny problem of the demarcation line between the didactic and the *adidactic* and, more generally, on the problem of the struggle between the *adidactization* of the world that the students must carry out and the *didactization* of the world that is the teacher's inexorable fate—a struggle on which we will say more a little further on.

An important aspect of the variant P_1^{+++} is the teacher y's *topos*. In the variant P_1^{++}, we see the teacher y multiplying the suggestions to X, often in the form of short, informative questions, which makes y look like a sphinx or a riddler and students like guessers. In the variant P_1^{+++}, on the contrary, the *topos* of y, during an "activity," consists in managing the activity, *not* its content, on which y does not intervene, but in its progress, in particular to recall the instructions which frame the activity. For the rest of their activity, students are referred to their interaction with the milieu.

[3] Since the highest antiquity, tables of inverses were available for this purpose.

[4] See "Dialectic of conjecture and proof (or dialectic of media and milieus)" in the glossary of the ATD (Chevallard, 2020, p. xxii).

In many respects, the variant P_1^{+++} is the framework of a *didactic revolution*, which upsets previous paradigmatic landscapes, and of which two key words are "adidacticity" and "milieu." We arrive here at the extreme point beyond which another revolution opens up, consubstantial with Paradigm 2, thematized by the *anthropological theory of the didactic* (ATD), and which extends the first revolution.

4.3 Paradigm 2 and its first variants

4.3.1 A new study format

The *primitive* variant of Paradigm 2, P_2^0, is characterized by the following fact: the class work starts from a question q on which the class *inquires*—the American expression *inquiry-based learning* has popularized this idea. From an epistemological point of view, P_2^0 is a regression with respect to P_1^{++} and P_1^{+++}: More often than not, behind the question q there is a work w that is the real learning stake and, when the didactic "set-up" is correctly designed, that is, for the class, the *missing* key element to answer question q. However, from a didactic point of view, P_2^0 is contemporary with the promotion of new forms of study—*inquiring*.

The emergence of Paradigm 2 first comes at a very high price. The formal requirement to investigate a question q leads to propose questions that may have only an improbable relationship with the work w, whose continued presence attests to the more or less surreptitious survival of Paradigm 1. The question q then tends to be the foil of w, which is still the main character. In the variant P_1^{+++}, given a work w, the choice of the question q is often difficult because it is subject to strong constraints, which we can summarize, by borrowing from the TDS, by saying that the question q must be able to be associated with a *milieu* allowing the creation of an *adidactic situation* (in fact, a sequence of milieus and a sequence of adidactic situations). In variant P_2^0, the formal requirement to "start from a question" leads to abandon these requirements and return to the guessing game traditional in the variant P_1^{++}. However, this oblivion of the dialectic of didactization and adidactization seems to have been the condition of entry, often unnoticed, into Paradigm 2.

4.3.2 An example of a study and research activity

The formal break initiated in the variant P_2^0 will be deepened in the framework of a variant that we will denote by P_2^+. First, the notion of a *study and research activity* (SRA) is defined, consisting of the study of a question q which, conducted under certain conditions and constraints, will cause a normally predetermined work w to be encountered. Here is a "simple" example: How can the product x of two lengths a and b be determined graphically (with ruler and compass)? This is question q. In studying q, a first step is to note that the equality $x = a \times b$ is not dimensionally homogeneous (a length equal to a *product* of lengths). We therefore rewrite it as $1 \times x = a \times b$. If no theorem is known whose conclusion contains an equality of *products* of lengths, and if a search in the readily available literature has proved fruitless, one can rewrite the previous equality as an equality of *ratios*, for example, $x / b = a / 1$. A search of the elementary geometric literature leads to the conjecture that Thales' intercept theorem (see e.g., Ostermann & Wanner, 2012, pp. 3–5) might be the key to the answer a sought. In fact, consider Figure 1 below, where the lines (BC) and (MN) are parallel and where we have AM = a, AB = 1, BC = b.

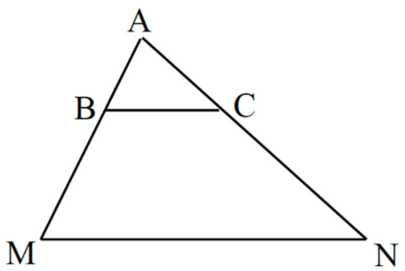


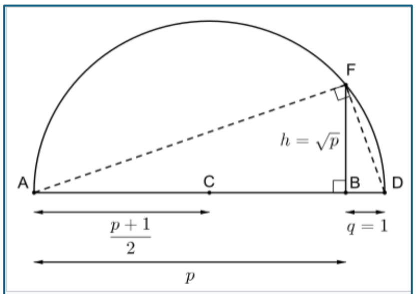
Figure 1. Theorem of the points in a triangle

According to the intercept theorem, we have: MN / BC = AM / AB. Let us set x = MN; we then have: $x / b = a / 1$, QED. Here, the "useful work" w is thus—classically—the "side splitter" theorem (see e.g., Side Splitter Theorem, n.d.).

4.3.3 SRAs and SRPs

The preceding example illustrates, as we will see, the starting point for the theory of SRAs and *Study and research paths* (SRP) developed in the ATD and suggests some of the major problems raised by the development of this theory. Thus, what was specified in the treatment of the previous graphical calculation problem was based on the assumption that the students would have already reached a fairly low degree of didactic dependence on the teacher in the domain considered. For example, one can imagine very simply that they have a geometry textbook containing a large number of theorems and that, initially, their research, in the construction of the milieu M of the study, is limited to this corpus of works, without specific guidance from the teacher. At the same time, one can imagine that the teacher, exceeding their role as manager of the activity, dispenses a more or less sibylline "advice" ("Here, there is a 'famous' theorem that will help you..."; etc.). We are then again in front of the struggle mentioned above between didactization and adidactization. Let us note that, in this struggle, students and teachers can be on one side as well as the other. The teacher may tend to constantly didacticize the situations experienced, but at the same time the students may demand more and more didacticity ("Teacher, but which theorem? Pythagoras?...") Conversely, the teacher can also seek to help students decrease their dependence on the heterodidactic, that is to say didactic help from *others* (and in particular from the teacher, seen as the "master of knowledge") in order to focus on the autodidactic and thus increase their didactic autonomy.

The SRA for the product of two lengths is said to be "finalized" in the sense that it has a purpose (an end): to make students encounter the work w. We will denote such an SRA by SRA_f. But then we can observe that the proposed question q is a specimen of a question *schema* Q that can be stated as follows: How can one determine graphically (with ruler and compass) the value of $f(a, b, c, ...)$ where $a, b, c, ...$ are given lengths and f is a given function? After the question $q_1 = q$ one may for example propose to study the question q_2 where $f(a, b) = a / b$ or the question q_3 where $f(a) = 1 / b$, or the question q_4 where $f(a) = \sqrt{a}$, etc. It is left to the reader to verify that the study of q_2 or q_3 will make one encounter—under the "usual" conditions and constraints—the work w_1 already encountered when studying q_1: Thales' theorem. But consider the question q_4, that is, the construction of the length $\sqrt{a}$. A cursory exploration of the literature yields, for example, the construction shown in Figure 2, borrowed from the Wikipedia article "Geometric Mean Theorem" (2025)—which solves the problem.



Construction of √p by setting q to 1

Figure 2. Illustration of the Geometric Mean Theorem

We thus discover a second work, w_2, relevant in the study of Q: the height h in a right-angled triangle is the geometric mean of the segments p and q that it creates on the hypotenuse, that is, $h^2 = p \times q$ or $h = \sqrt{p \times q}$. The expected result is obtained by taking $p = a$ and $q = 1$.

4.4 The notion of inquiry and the definalization process

4.4.1 Definalizing SRPs

We can now, cautiously, take stock of the initial notion of SRP. Let a question schema $Q = \{q_1, q_2, q_3, ..., q_n, ...\}$ be put under study in a class $[X, y]$. The decision to study Q rests with y, after a possible debate in $[X, y]$ ("Shall we study Q or Q' or Q?", for example). The order in which the class addresses the questions $q_l \in Q$ (some of which will possibly be dropped) is proposed by y with X's apparent agreement. This done, it is up to X, under the direction of y, to constitute and modify (by increasing it, lightening it, etc.), throughout the duration of the study, the milieu M, which will contain the works w_j which the class $[X, y]$ will have used or would consider using. From this point of view, the semi-developed Herbartian schema

$$[S(X, y, q_i) ➡ M] ➥ a^{♥}$$

should be parameterized by the time of study t, so that we will have: $[S(X, y, q_i)$ ➡ $M_t]$ ➥ $a_t^{♥}$.[5] Add to this that, during the same class session (say, in a two-hour class session), the class will be able to study (partially) two or even three questions, the study of which is unevenly advanced—again, the final decision is up to y.

Moving from the isolated SRA to the SRP as described so far moves us from the variant P_2^{+} to the variant P_2^{++} of Paradigm 2. The finalization of the study of a question q by a specific work w is succeeded by a finalization by a whole set $\mathcal{W}$ of works w— for example all those present in the class geometry textbook or in the geometry articles of the Wikipedia encyclopedia. There then tends to be, for the questions $q \in Q$ involved, a certain *definalization* of their study. We will formulate here the hypothesis that definalization is a factor (among several others) of adidactization of the study, notably because it makes it more uncertain for the student to question the didactic intentions of the teacher formulated in terms of works to be visited.

4.4.2 Building an unheralded, appropriate milieu

The definalization is all the more pronounced as the search for works w that prove to be effective tools for answering question q may lead to works not listed (or that have ceased to be listed) in the traditional curriculum. Let's go back to the question q_1 of the product of two lengths a and b. In a first stage of the inquiry, we assumed that no "known" theorem referred to the *product* of lengths, and that only *ratios* of lengths could be found in the explored geometric corpus. However, let us suppose that, in a second stage of inquiry, we discover a property concerning products of lengths, namely the theorem defining what is called *the power of a point with respect to a circle* (see "Power of a Point," 2025), which Figure 3 illustrates: the (arithmetic) "power" of the point P is

[5] The downward arrow ➡ can be read "creates" and the upward arrow ➥ "generates."

the product PA × PB, which is constant for any line chosen through P that intersects the circle in A and B. Thus, we have PA × PB = PU × PX. If we choose U such that PU = 1, and A and B such that PA = a and PB = b, then the length x = PX satisfies $1 \times x = a \times b$, which answers (in a different way) the question q_1 (provided we know how to construct the circle through three non-aligned points A, B, U).

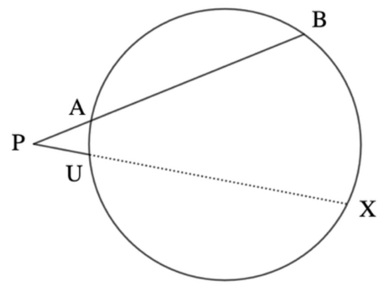


Figure 3. Configuration for PA × PB = PU × PX

This same work w'_1 also answers question q_4 about the construction of $\sqrt{a}$: when, in Figure 4, the line (PT) is tangent to the circle, we have PU × PA = PT^2. If PU = 1, PA = a, and x = PT, then $x^2 = 1 \times a = a$ and thus $x = \sqrt{a}$[6].

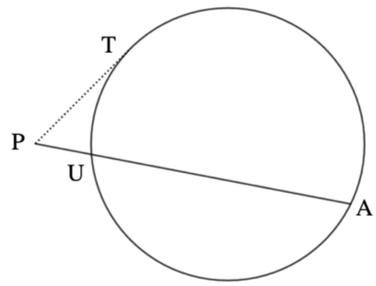


Figure 4. Configuration for PT^2 = PU × PA

This definalization of inquiries allows, more generally, for the "sorting" of works present in a given curriculum, bringing to light works that are "absent" from the curriculum but yet useful and, on the contrary, works that are never used to answer the questions q one is asking. This is an important mechanism in the process of *curricular redefinition* that can combat curricular aging and congealment.

4.5 Paradigm 2 and didactic organizations

4.5.1 Basic principles of pedagogic organizations

From the variant P_2^{++} will emerge a variant which we will denote by P_2^{+++}, which will lead to raise the main current (and future) problems of the transition from Paradigm 1 to Paradigm 2. In what follows, we will refer to a family of *didactic organizations* whose basic structure we will describe, while often giving possible examples chosen for their unexpectedness and thus, in general, unconsciously dismissed.

Let there be a class $c = [X, Y]$, where X is the set of students and Y is the set of teachers of the class. This class is assumed to have three kinds of groupings: (a) first of all, the *general meeting* of the class, where, periodically, all the students X and teachers Y meet to review the work done by the class and confirm, adjust or extend its work program for the current or future period; (b) then, the *seminar*, which we will come back to; (c) finally, *thematic workshops*, which may bring together students from several classes of the school, and which we will also come back to. We will see later that the seminar is the very heart of the work of the class.

4.5.2 Basic principles of didactic organizations

The governing principle of the class c's activity is the *study of* or *inquiry into* so-called *generating questions* q. (The origin of these questions will be examined later.) Once the study of such a question q has been decided, in one or more general class meetings, a didactic system $S_q = S(X_q, Y_q, q)$, where $X_q \subseteq X$ and $Y_q \subseteq Y$ is formed (in the framework of the seminar). Note that, if y_μ is the math teacher, we can have for example $Y_q = \{y_\mu\}$. The goal of S_q is to *inquire into* q and submit regular *progress reports* in the class seminar, presenting for validation the state of the answer a under construction. The *final report* on the study of q, once duly

[6] Note that in Figure 3, the dotted segment UX marks an auxiliary secant introduced solely for comparison. In Figure 4, the dotted segment PT designates (a part of) the tangent produced by the construction itself. Thus, dotting serves two functions: it highlights a non-essential auxiliary element in the first case and a constructed, non-given element in the second.

approved by the class, will be one of the chapters in the *class book* of $\mathscr{c}$ for that year. The major rubrics of this final report will be clarified later. But note that, in addition to presenting and analyzing the class's final answer, $\mathscr{a}^{\heartsuit}$, it will need to present and analyze the questions generated by the study of $\mathscr{q}$, the answers to them, and the works $\mathscr{w}$ used. The *in-class exam* will cover all or parts of the class book.

Let us return to the semi-developed Herbartian schema $[S(X_{\mathscr{q}}, Y_{\mathscr{q}}, \mathscr{q}) \Rightarrow M] \Rightarrow \mathscr{a}^{\heartsuit}$. In variant P_2^{+++}, given a question $\mathscr{q}$ to investigate, the didactic system $\mathscr{S}_{\mathscr{q}} = S(X_{\mathscr{q}}, Y_{\mathscr{q}}, \mathscr{q})$ has two closely related tasks to perform: the first, which is a means, is the continued construction of the milieu M; the second, which is the end, is the construction of the final answer $\mathscr{a}^{\heartsuit}$. In principle, saying that $\mathscr{S}_{\mathscr{q}}$ constructs M means that $X_{\mathscr{q}} \cup Y_{\mathscr{q}}$ constructs M. In the previous variants, especially in the variant P_1^{+++}, it seems that the construction of M is a prerogative of $Y_{\mathscr{q}}$, to which $X_{\mathscr{q}}$ has little share. By constructing a supposedly adidactic milieu, teachers perform an essential part of their function: instead of teaching students directly, they do so by the mediation of the milieu. By contrast, a distinctive feature of the variant P_2^{+++} is that the construction of M falls to $X_{\mathscr{q}}$—as always, under the responsibility of $Y_{\mathscr{q}}$ (who may ask $X_{\mathscr{q}}$ to justify, to some extent at least, their choices). This topogenetic change is an essential didactic fact.

4.6 Didacticity and adidactization

4.6.1 A new topos for the student

The topogenetic change in question drastically reduces the *topos* of $Y_{\mathscr{q}}$ and correlatively increases the *topos* of $X_{\mathscr{q}}$ in which a type of tasks appears that is new to the students, which the teachers previously carried out outside their presence—while preparing, sometimes at length, their teaching. Let us note at this point that what we called above the process of adidactization actually affects the relation of the student x to a work $\mathscr{w}$: it is this relation $R(x, \mathscr{w})$ that is didactized with respect to y. This means that x gives up believing that the works $\mathscr{w}'$ composing M would carry within themselves, in a cryptic way, suggestions, or even injunctions from y, which x should strive to identify. It may however happen that, on the contrary, the relation $R(x, \mathscr{w})$ gets more "didactized", in the sense that x sinks into hoping that the composition of M has in it some surreptitious hints from y. That said, it is reasonable to think that the indicated redefinition of the student's *topos* is, in a general way, likely to favor the adidactization of their relations to the works concerned.

4.6.2 Inquiring and adidactizing: A nontraditional example

Let us dwell on an example. Consider the following question $\mathscr{q}$: "A farmer needs to ship a batch of 250 eggs in boxes that can each hold 6 eggs (as exemplified in Figure 5). How many boxes does he need?"[7] A teacher y asks a student x to solve this problem. What relation can x have to the situation $\mathscr{s}$ thus created? (The word "situation" is understood, in the ATD, to mean the *state* at a given moment $\mathscr{t}$ of a complex of institutions, institutional positions and persons subjected to these positions.)

Figure 5. Boxes with a capacity of 6 eggs each

Before considering this question, imagine that student x is replaced by an employee $\bar{x}$ of a company hired to perform the calculations necessary to run the business, and y is replaced by the head $\bar{y}$ of $\bar{x}$. So $\bar{y}$ asks $\bar{x}$ to determine the number of boxes of 6 eggs needed to transport 250 eggs. This gesture of $\bar{y}$ is not looked at by $\bar{x}$ as a didactic gesture, but without doubt as a *non-didactic* gesture. Let us note that the circumstances may be such that $\bar{x}$ is wrong, in the following sense: $\bar{y}$ actually looks at his gesture as didactic with respect to $\bar{x}$ regarding this or that work $\mathscr{w}$. (From the point of view of the ATD, *any* gesture can be considered as didactic by certain instances.) That noted, it may be for $\bar{x}$ a task of a type he performs often and routinely: $\bar{x}$ quickly responds to $\bar{y}$ that, in the case under consideration, he will need 42 boxes.

If we then return to x and y, it may well be that x has the same relation to the question $\mathscr{q}$ that we lent to $\bar{x}$. Generally speaking, the relation $R(x, \mathscr{q})$ depends on the cognitive and praxeological equipments of x, on the milieu M that x will be able to constitute, on the attitude of the teacher y, and, more generally, on the ambient situation. All these conditions and constraints, which differ from one situation to another, can generate quite different relations $R(x, \mathscr{q})$. For example, the question $\mathscr{q}$ posed by y to x may appear to x as a riddle, which x thinks he can only answer on a guess: "20 boxes? 30?

[7] We will use the masculine gender to refer to a generic person.

More?...” Instead, the proposed question will appear to $\bar{x}$ as a task of a well-known type, for which he has a routine technique. If, for example, it is required to carry 3250 identical objects in boxes of capacity 18 (see Figure 6), $\bar{x}$ calculates the quotient 3250 / 18 using the Word calculator, $\mathbb{w}$, which displays the following result: 180,555555556; $\bar{x}$ concludes that the answer is 181. However, for a lower secondary education student, this type of tasks does not necessarily exist: what exists, in general, is the type of which a “representative” specimen would be to determine how many people can be given 6 objects (6 candies, etc.) if 250 of these objects are available. Therefore, the type of tasks associated with $\mathscr{q}$ must come into existence and a suitable technique must be created. This is the problem—which, as we will see below, is akin to, but different from the much more popular problem of dividing integers.

Figure 6. A box with a capacity of 18 eggs

Here is a case observed several times in a ninth-grade class: Students respond to $\mathscr{q}$ by proposing one of the *non-whole* numbers 41.6, 41.66 or 41.666, etc., obtained with their calculator $\mathbb{c}$. (For example, we have for $\mathbb{c} = \mathbb{w}$: $250 / 6 =_{\mathbb{w}} 41.666666667$.) It is as if these students were unaware of the question asked, which requires a whole number as an answer. Their relation $R(x, \mathscr{q})$ is didacticized by the fact that their answer is expected by y, whose expectation they interpret and seek to satisfy by showing, here, that they “know” how to do a division. Here, this work $\mathscr{w} = \mathscr{d}$ which is the division operation, denoted by a / b, is powerfully attractive, unlike the operation we would denote today by $\lceil a / b \rceil$, which remains largely unknown in elementary school culture—where one equally distributes items between “instances” but apparently does not have to carry them from one place to another! Note that the milieu M is then essentially reduced to $\mathscr{d}$ and the calculator $\mathbb{c}$ which, itself, may be more or less strongly didacticized: if the given result is contested (for example by y), some students may be tempted to defend themselves by invoking the calculator as if what it asserts (or rather displays) relieved them of their epistemic responsibility, just as they would defend themselves by saying: “It was the teacher who wanted that!”

We will now pose a hypothesis that complements the previous one about the role of the study’s definalization in the process of adidactizing the relations $R(x, \mathscr{q})$: Providing students with a “ready-made” milieu M does not only foster the didactization of $R(x, \mathscr{q})$, but the process is even reinforced as students are more likely to follow a prescribed roadmap in their interactions with M. The temptation to “direct” students closely is a special case of the temptation that “superiors” in any institution may feel to control closely the actions of those they regard as subordinates, or even to algorithmize their actions so that they become more predictable and controllable. For example, in matter of didactic engineering, there may be a temptation to design products that one hopes will prove “teacher-proof” or even “student-proof.” In contrast to this temptation to “over-direct” the students, the notion of *inquiry* developed in the ATD requires that the class *takes the time* to investigate and investigates more *freely* (with respect to the construction of the milieu M and its use) than is customary in traditional schoolwork.

4.7 A different didactic world

4.7.1 A bit of a seminar session

Let us imagine what could happen if students were freer to do as they please. A student x_1 begins by considering that, by taking 6 boxes each time, he can carry first $6 \times 6 = 36$ eggs, then 36 eggs again, which makes 72 eggs, to which are added 36 eggs, that is to say in all 108 eggs, increased again by 36 eggs, that is to say... 144 eggs. Seeing that the batch of 250 eggs is still far away and not knowing where he stands regarding the number of boxes, he stops here: for him, the inquiry has started but seems to have come to an end. A student x_2 does this differently. Since $6 \times 8 = 48$, therefore 8 boxes will carry 48 eggs, so there are 202 eggs left to carry. Since $6 \times 7 = 42$, therefore 7 new boxes will leave 160 eggs to carry. With 10 new boxes, there will be only 100 eggs left to carry. With another 10 boxes, there are only 40 eggs left. With 6 new boxes, there will still be $40 - 36 = 4$ eggs to transport. This time, x_2 has the number of boxes used: $8 + 7 + 10 + 10 + 6 = 41$ boxes; at the same time, he discovers that there are still eggs to be transported: to transport these 4 eggs, one more box is needed, which will make 42 boxes in all. This result, which seems promising, is presented in the Seminar. But is it “solid”? A student, x_3, points out that $6 \times 42 = 252$ and thus 42 boxes can carry all 250 eggs, whereas, since $6 \times 41 = 246$, therefore 41 boxes cannot carry 250 eggs. The Seminar agrees. A student, x_4, suggests to first consider, not 8 boxes or 10 boxes, but “directly” 40 boxes, which makes it possible to transport 240 eggs, so that there remain ten eggs to be transported, that is to say 6 + 4, which still requires two boxes: one finds thus the 42 boxes already calculated... Here the teacher asks x_4 how he arrived “directly” at 40 boxes; the student replies that he used the first result shown in the Seminar. The teacher y asks, “What if, instead of 250, we have 115 eggs to carry?” The student x_4 seems to hesitate; y then notes that, as a discovery technique, what x_4 proposes does not really advance the inquiry. Can we do “better” than what we have done so far? Another student, x_5, says, “Teacher, can I use a spreadsheet?”

The teacher: “You can use a spreadsheet in the sense that you are *allowed* to do so: here you only have to give a reasonable justification for everything you do. Whether you are *able* to use a spreadsheet, I guess, depends on what operations you want to do with...” The student states that he would like to have the multiples of 6 displayed until they exceed 250. Teacher: “That’s an idea. The inquiry continues. See you all at the next Seminar session!”

4.7.2 The seminar goes on

We can still imagine that, as the Seminar continues, the class arrives at a first technique, τ_1, using a spreadsheet. To calculate the number of boxes, we choose a multiple of 6 “close” to the number n of eggs to be transported. For example, if $n = 115$, this might be $66 = 11 \times 6$; if $n = 250$, we might choose $300 = 50 \times 6$. We then arrive at the left-hand block of Table 1 (where the first two columns increase downward, while the last two decrease).

11	**66**		**50**	**300**
12	72		49	294
13	78		48	288
14	84		47	282
15	90		46	276
16	96		45	270
17	102		44	264
18	108		43	258
19	114		42	252
20	120		41	246
21	126		40	240
22	132		39	234

190	3420
189	3402
188	3384
187	3366
186	3348
185	3330
184	3312
183	3294
182	3276
181	3258
180	3240
179	3222

Table 1. Values obtained by choosing multiples of m close to n for $m = 6$ (left block) and $m = 18$ (right block)

All this applies when, more generally, the boxes are of m eggs, for example $m = 18$. For $n = 3250$ eggs, starting from $3420 = 190 \times 18$, we have what we see in the right-hand block of Table 1. Through various numerical tests, the investigation can then deliver the following conclusions: (a) if we denote by $B_m(n)$ the number of boxes of m eggs needed to transport n eggs, $B_m(n)$ is the smallest multiple of m greater than or equal to n; (b) if we call the function that assigns to any number x in the interval $(k, k + 1]$ the value $\lceil x \rceil = k + 1$ a *ceiling* function, we have $B_m(n) = \lceil n / m \rceil$ (see “Floor and Ceiling Functions,” 2025). The *Big Online Calculator*, which can be found at https://www.ttmath.org/online_calculator, and which designates the ceiling function by “ceil,” gives the result shown in Figure 7.

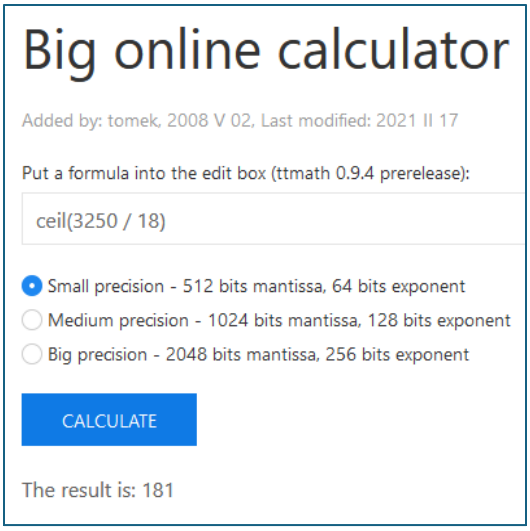


Figure 7. Output from the Big Online Calculator

We thus obtain a new technique, τ_2, a variant of which, τ_2', can, as we have seen, do away with the ceiling function, at the risk of some people falling into the “didactic” trap where we have seen ninth graders fall.

4.7.3 Deepening and completing our exemplary study

When writing the chapter (or subchapter) of the class book featuring the study of question q, final edits can be made. Thus, with respect to the τ_1 technique, we may, for example, specify what we have called the “choice” of the multiple of m from which to determine $B_m(n)$. Suppose we want to determine $B_{37}(8912)$. This integer must not be much different from 10000 / 40, or 250. We have $37 \times 250 = 9250$. Taking successive decreasing multiples of 37 starting at 37×250, we arrive at $B_{37}(8912) = 241$ (see Table 2). (In fact, we have: 8912 / 37 =w 240.864864865.) We will also see the evolution of the difference $n - mk$: the 241st box will have 5 free spaces (and will therefore contain only 32 eggs).

k	$37k$	$37k$ - 8912
250	**9250**	338
249	9213	301
248	9176	264
247	9139	227
246	9102	190
245	9065	153
244	9028	116
243	8991	79
242	8954	42
241	8917	5
240	8880	-32
239	8843	-69

Table 2. Values of $37k$ and the differences $37k - 8912$ for decreasing integers k, leading to the determination $B_{37}(8912) = 241$

Now suppose we want to determine $B_{42}(11057)$. Again, this integer must not be much different from 10000 / 40, or 250. We have $42 \times 250 = 10500$. Taking successive increasing multiples of 42, we arrive at $B_{42}(11057) = 264$, as shown in Table 3. (In fact, we have: 11057 / 42 =₩ 263.261904762) One will also see the evolution of the difference $mk - n$: the 264th box will have 31 free spaces (and will therefore contain only 11 eggs).

k	$42k$	$42k$ - 11057
250	**10500**	-557
251	10542	-515
252	10584	-473
253	10626	-431
254	10668	-389
255	10710	-347
256	10752	-305
257	10794	-263
258	10836	-221
259	10878	-179
260	10920	-137
261	10962	-95
262	11004	-53
263	11046	-11
264	11088	31
265	11130	73

Table 3. Values of $42k$ and the differences $42k - 11057$ for increasing integers k, leading to the determination $B_{42}(11057) = 264$

It will have been noted that the technique used here dispenses with the division operation. The above further draws attention to the (non-traditional) system $n = m\tilde{q} - \tilde{r}$, $0 < \tilde{r} \leq m$, where $\tilde{r}$ is the number of places left empty in the $\tilde{q}$-th box (if $\tilde{r} = m$, this "last" box is not used). The report on the study of question q should contain not only the precise answer $A^{\heartsuit}$ arrived at by the class $[X, y]$, but also useful indications on all the works used or encountered, without yielding to the temptation of an "exhaustive" study of these works—with regard to the spreadsheet used, for example, only the indispensable manipulations should be specified, without claiming to "study (exhaustively) the spreadsheet used." That noted, this is perhaps a good time to emphasize an important fact: any inquiry must bring to those who are carrying it out the feeling that their relations to some of the objects encountered in the course of that inquiry are changing—and in particular that "new" knowledge, for X but also, sometimes, for y, is thus being uncovered, even when working on a question q regarded as "elementary" or "well known" by some people and institutions whose "certainties" this very inquiry may well challenge. This is also true, of course, for the researcher ξ.

In the case of the ninth graders x_i who answered that the number of 6-egg cartons needed to transport 250 eggs was *non-integer*, their relation $R(x_i, q)$ was didacticized by the fact that their answer was expected by y, whose expectation *they* interpreted in their own *didactic* way and sought to satisfy by showing, here, that they "knew" how to do a division! We can generalize this conclusion by formalizing it somewhat. Let $\mathcal{S}_q = S(X, y, q)$. We will denote by *degree of didactization* of the relation $R(x, q)$, where $x \in X$, the degree to which x is dependent on the relation $R(x, R(y, \mathcal{S}_q))$, that is to say the relation of x to the relation of y to

$\mathcal{S}_q = S(X, y, q)$. But beware! It is not abnormal that $R(x, q)$ is constructed by taking into account $R(y, q)$ (provided of course $R(x, R(y, q)) \neq \emptyset$). In this sense, $R(x, q)$ is dependent on $R(x, R(y, q))$—for example if one of the answers $a^\diamond$ is in fact the answer $a_y^\heartsuit$. More generally, if the person z (who may be a classmate, a relative, etc.) is the author of some answer $a^\diamond$, then $R(x, q)$ will (partially) depend on $R(z, q)$ through the relation $R(x, R(z, q))$. Let us repeat here that the success of the inquiry into q is therefore based on the overcoming of the dialectic tension between the adidactization that x must constantly seek (and of which the process of definalization of the study of q is a key factor) and the didactization that y necessarily achieves in fulfilling the teacher's mission.

4.8 The question of questions

4.8.1 A crucial difference

We return here to what we call the "question of questions": *Which* questions q may be studied? The essential—but not the only—principle that distinguishes Paradigm 2 from Paradigm 1 in this regard can be presented as follows. In Paradigm 1, the teacher y (or the didactic engineer ξ^*) starts with a work w to be taught (because it is duly included in the "official" curriculum) and then seeks a question q whose study, under certain conditions and constraints, must lead the class $[X, y]$ to encounter, use, and study as much as is relevant to the ongoing inquiry, the work w. In Paradigm 2, the class studies, under certain conditions and constraints, a question q chosen (by a certain instance $\hat{\imath}$) for its formative relevance (we will come back to this notion) and one discovers in the course of the inquiry into q the works w_1, w_2, w_3, ..., which the class will study as much as it seems necessary to be able to use these works adequately. One can think that the paradigm of visiting works tends to generate a relation $R(x, q)$ to the proposed question that is potentially didacticized by the question "What work w does the teacher want us to encounter and study by proposing this question?" This systemic suspicion disappears, or at least lessens, in Paradigm 2. "What do you want to teach us?" asks the suspicious student x. "Nothing," replies y. "It's about providing an answer a to the question q," he continues. And adds, "It's up to us to see how…"

4.8.2 A naïve, open question posed to the class

We will start with a few simple but important remarks. Our first remark is this: a question q to be studied is most often a "naïve" question or at least will appear so after the inquiry into it is accomplished. This is true in all cases: before becoming experts, researchers are novices if the question they are studying is really an open question in their field of expertise.

Our second remark is this: any question q, no matter how small, can lead to a meaningful inquiry, productive not only of a useful answer a but also of previously unexplored knowledge. The question of eggs to be transported is, from this point of view, a telling example, to which we will not return here.

Whatever the instance $\hat{\imath}$ which performs it, the choice of a question q that a class $[X, y]$ will have to study should not be restricted to the questions that these students "ask themselves." It should be extended in priority to questions *they have to face*—without necessarily being aware of it. Questioning the world requires a realistic ethics that fights against the "self-confinement" provoked by ordinary existential egotism. Even more broadly, to help a student x come to occupy the position of the person designated by $\bar{x}$ above, and y that of $\bar{y}$, one can create, as part of the Seminar, a "consulting and research unit" receiving orders from "clients" (denoted by $\hat{\imath}$ above). It should be noted that the corpus of traditional arithmetic problems is based on such an organization supposed to manage the relations between the inside and the outside (more or less imaginary) of the classroom. This is, once again, what the question of the eggs reminds us of: it is not the student who is supposed to have to carry 250 eggs, but "a farmer," who is the "client" of the class's consulting and research unit—an imaginary, "objective actor," located outside the classroom. There are thus the questions that x asks himself, those that x has to face (whether x likes it or not), and those that other people have to face: we thus pass from the individual to society.

4.8.3 A maze of questions

In order to rebuild educational systems around the world, we must conduct a never-ending inquiry into these three (non-disjointed) categories of questions: those that these students ask themselves, those that weigh on them without their being aware of it, and those that are posed to other people and to which the students must help the world around them to answer. This permanent inquiry is a collective, collaborative project, about which we will make only a few observations here.

Regardless of the instance $\hat{\imath}$ that chooses a question q, the criteria for choice should not include the fact that the study of q would lead the class to encounter this or that work in the (still) existing w-curriculum (typical of the paradigm of visiting works), which it would be precisely a matter of "rejuvenating" by making it more adequate to the cognitive and praxeological needs experienced in the set of inquiries conducted. Quite to the contrary, one must resolutely ignore, at the outset, a question that would only arise in the inquiry into question q, and which is therefore a question q_k generated by the inquiry and will be considered in its own time. To do this, it is appropriate to allow for the case where, studying a question q, the class encounters a work w that appears to be an appropriate study tool but that also seems clearly out of reach given the average praxeological equipment of the class—which is by no means to say that the inquiry should be abandoned. One can think for example about the case of a ninth-grade class investigating the basic reproduction number R_0 of an epidemic—it will be possible for this class to master the elementary formula $P = 1 - 1/R_0$, which gives the minimum proportion P within a population that must be immunized to prevent the development of an epidemic. But the class will probably not be able to go any further (see e.g., Jones, 2007).

Generally speaking, a question q should be studied by looking at it as a question *generated* by the (actual or potential) study of a "overhanging" question $\bar{q}$, which specifies the problematic of the study of q. For the egg question, thus, the overhanging question $\bar{q}$ might be, for example, $\bar{q}_1$ = "What are the mathematical problems of transporting goods and how do you solve them?" or else $\bar{q}_2$ = "What are the mathematical problems of everyday life and how do we solve them?" Depending on whether q is considered part of the study of $\bar{q}_1$ or $\bar{q}_2$, its study may be pursued differently, under the responsibility of the instance in charge of the conduct of the inquiry (in a classroom, it is more often than not the teacher). The application of this principle must, however, be limited to questions $\bar{q}$ that are not, under the prevailing conditions, too highly polemical: for the intensity of the debate could then hinder, by simplifying or even blocking it, the study process, for example by imposing ready-made "answers" that would not be adequately analyzed and put to the test of the dialectic of media and milieus.

4.9 Which teachers are needed to question the world?

4.9.1 The strange make-up of anyone's knowledge

Studying a question q as if it were a question generated by the study of an overhanging generating question $\bar{q}$ may involve only one domain of knowledge, however broad it may be—for example, mathematics. Yet, quite often, such a question $\bar{q}$ spontaneously refers to several more or less extensive fields of knowledge (physics, biology, history, etc.). This is likely to be the case if, for example, $\bar{q}$ is the overhanging question "What causes global warming?" and q is the question "How does the rise in average temperature depend on the level of CO_2 in the atmosphere?" From the class $[X, y]$, or rather $[X, y_\mu]$, we then move to the class with its set Y of teachers, that is, $[X, Y]$. The inquiry into q will be conducted by a didactic system that can be written, as we have seen, $\mathcal{S}_q = S(X_q, Y_q, q)$, where $X_q \subseteq X$ and $Y_q \subseteq Y$. Here, Y_q should ideally bring together, in the course of the inquiry, a "specialist" from each of the disciplines $\mathcal{D}$ that will be involved in the study of q. Is that possible?

Before answering, let us observe that what is expected of the student team X_q is, not that it is, but that it *becomes*, as a collective, an "expert" (to some degree), not in mathematics, physics, biology, history, etc., *in general*, but on the *questions* of mathematics, physics, etc., that X_q will encounter in the investigation into q. The same logic should apply to Y_q as a collective: when the team of teachers Y_q leads the investigation of q, they have to make themselves "experts", in the wake of the team of students X_q, on multiple questions that the members of Y_q have never before investigated, either because these questions are not within the competence of "their" discipline, or because they have never before encountered them in this setting. We will give here one of the simplest possible examples concerning the praxeological equipment of secondary school mathematics teachers. The formula $x = \frac{2c}{-b \pm \sqrt{b^2 - 4ac}}$ will remind them of the well-known expression for the roots of the quadratic equation $ax^2 + bx + c = 0$, which is usually written $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. But what is this formula? Who uses it, and why? Most teachers of today, it seems, cannot answer these questions—which we will leave to the reader's perspicacity.

The comparison of "teachers" and "students" just made may shock the reader who is subjected to an institution where the distinction of these positions is based on an *intrinsic* distinction between teachers and students—the teachers know, the students do not know (yet) —, instead of a *functional* distinction—the teacher teaches, the student learns, or rather: the student studies to learn, the teacher helps the student to study—and, to do so, the teacher must have learned and must often study again to continue learning. To combat the intrinsic vision traditional in the paradigm of visiting works, one can consider, for example, the case of a ninth grader tutoring a sixth grader: the first acts as "teacher," the second as "student"—there as elsewhere the fiction prevails that the teacher "knows," and that the student has to learn. One can also consider the case of a training course for a group of teachers X where the trainer y is a "specialist" from the neighboring university—who, sometimes, will find that these secondary school teachers are quite ignorant on points that this trainer thought they should be aware of. But let us observe this: If, by some twist of fate, this university specialist, who we assume has a PhD in mathematics, were to find himself teaching mathematics to high school students the following week, he would likely have to work hard to learn a lot of things that are foreign to him, including in mathematics!

More generally, it is necessary to take a lucid, realistic look at what is usually the praxeological equipment, and more particularly the mathematical equipment of a secondary school teacher—but what follows applies in all cases (from elementary school to university). Each person's mathematical equipment depends on the conditions and constraints under which that equipment has formed and continues to change and develop. For example, in the "classical" variants of Paradigm 1, the teacher is expected to be able to give an "appropriate" presentation for each of the works w designated by the official curriculum: strictly speaking, the teacher knows, not "mathematics," but presentations on mathematical works. It is easy to imagine, however, that the teacher's repertoire of "lessons" will evolve over time, for example because some works fall into disuse or some of their "properties" are surreptitiously discarded by the passage of time. The same remark is even more valid concerning, in the "modern" variants of Paradigm 1, the repertoire of "activities" composing the ideal mathematical equipment of the teacher position. This repertoire takes a long time to emerge; and, when this process has been completed, the result is usually partially obsolete. The passage of time is both destructive and, at its best, partially regenerative. Any praxeological equipment can appear as a strange medley of works often stripped of some of their parts. Here is a very simple example: Every mathematics teacher knows (and can teach) that the diagonals of a parallelogram bisect each other. The theorem according to which this property characterizes parallelograms is a well-known mathematical work. However, it seems that, today, many teachers would not necessarily be able to say right away what this theorem is (or was) useful for and why.

4.9.2 Which relation to knowledge and ignorance?

How does the cognitive and praxeological equipment of the teacher position in Paradigm 2 differ from that of the teacher position in Paradigm 1? The fundamental, more general distinction has to do with the relation that an instance $\hat{\imath}$ can have to knowledge (and thus to ignorance), a relation that we will denote by $R(\hat{\imath}, \mathrm{E})$, where $\hat{\imath}$ is a person or an institutional position—the letter E being the Greek capital epsilon, by which we denote here knowledge (in Greek, ἐπιστήμη, *episteme*). We will (partially) analyze $R(\hat{\imath}, \mathrm{E})$ in terms of *attitudes*: the ideal subject of Paradigm 2, whether teacher or student, *or other*, is defined by five attitudes. The first, $\hbar_1$, is the *problem finding attitude*, which consists in recognizing the "problematicity" of the situations experienced or observed, which means in raising questions about them. This is obviously an essential attitude, from which both the question q generating the inquiry and the generated questions q_k derive. The second relevant, even *sine qua non*, attitude is the *Herbartian attitude* $\hbar_2$, which consists in not running away from any question q as such (by denying it, ignoring it, repressing it) and, concretely, in engaging in its study *hic et nunc* or, at least, in putting its study *on hold*. The third attitude is the *procognitive* attitude, $\hbar_3$, which is opposed to the *retrocognitive* attitude typical of Paradigm 1, which makes us "look back" towards the knowledge *acquired until then*, which by the way is sometimes dubious (when it has not been driven out of our memory), thus abandoning ourselves to the *retrocognitive reflex*. In contrast, the $\hbar_3$ attitude consists in projecting oneself *forward*, to go towards the knowledge that will prove useful for the current inquiry and that one may not have encountered *until now*, in a *procognitive* tension towards knowledge. Indeed, we will consider that, when approaching an "open" question (for ourselves, but often for a few other people as well), *it is not abnormal* to "know nothing," or to know little, about that question, or about the tools—answers $a_i^\lozenge$, other works w_j, questions q_k—that its study may require bringing into play. It is the *question* q that counts, not the fact that one has or does not have, at the time one begins its study, the relevant tools—the relevance of which can hardly be known in advance.

The fourth attitude, $\hbar_4$, is the *exoteric* attitude, which is opposed to the *esoteric delusion* of one who believes to know (almost) everything (at least in some area). On the contrary, the exoteric attitude consists in *always* looking at oneself as having to *study* in order to *learn* again or, already, to *verify* or to *question* what one *thinks* one knows. The exoteric attitude is based on two principles which must govern the personal or institutional relation to knowledge and ignorance: (1) one must allow oneself *to be ignorant* in *any* field (including, therefore, one's own field of "specialization"); (2) one must *refrain* from *not* confronting one's ignorance, whatever the field (even if it seems to us to be completely foreign), in order to progress as much as possible and as much as *it is useful*, towards a praxeological equipment adequate to the investigation that this equipment is supposed to serve.

The fifth attitude, $\hbar_5$, is that of the *ordinary encyclopedist*. It consists in seeing oneself as *not foreign* to the whole of the possible praxeological fields, even if it would be with a "degree of exotericity" close to zero, while constantly striving to make this degree of exotericity grow, as far as this is useful. To give a more concrete picture of this principle, we imagine that each instance $\hat{\imath}$ owns and manages an *encyclopedia* $\mathrm{E}(\hat{\imath})$ composed of *books* listing the praxeologies encountered in different domains of human activity: for any discipline or sub-discipline $\mathcal{D}$, for $\hat{\imath}$ this encyclopedia will contain one book of $\mathcal{D}$, denoted by $\mathrm{B}(\hat{\imath}, \mathcal{D})$, the letter B being the Greek capital beta (in Greek, βιβλίον, *biblion*, means "book"). $\mathrm{B}(\hat{\imath}, \mathcal{D})$ is often empty ($\mathrm{B}(\hat{\imath}, \mathcal{D}) = \emptyset$) or almost empty ($\mathrm{B}(\hat{\imath}, \mathcal{D}) \approx \emptyset$). The "proper management" of the books $\mathrm{B}(\hat{\imath}, \mathcal{D})$ is obviously an essential requirement for the proper conduct of the investigations to be carried out or supervised.

Generally speaking, to be in harmony with Paradigm 2, one should have (or develop) a relation to knowledge and ignorance organized by the five attitudes $\hbar_1$, $\hbar_2$, $\hbar_3$, $\hbar_4$, and $\hbar_5$, which means that one should be "problematizing," Herbartian, procognitive, exoteric, and an ordinary encyclopedist—rather than a "distinguished scholar." Within this framework, prospective or in-service teachers will need to develop their praxeological equipment in two directions. First, they have to learn to identify and formulate questions q (thus relying on the attitude $\hbar_1$), then to study them by identifying and studying the answers $a_i^\lozenge$ available, the works w_j useful, the questions q_k generated (therefore drawing on $\hbar_2$, $\hbar_3$, and $\hbar_4$), and finally, to report on all of this with clarity and conciseness (in agreement with $\hbar_5$). Second, they have to learn to initiate, to guide, and to bring to an end (at least provisionally) inquiries carried out, in this case, by students themselves. Let us insist on the fact that what a teacher must "know" is acquired by personally carrying out inquiries and by supervising inquiries carried out by students, not by the decontextualized study of works that the history of school systems has made temporarily prestigious. As a final example, we will propose this question (of a monodisciplinary, rather than codisciplinary, type): "Is a derivative function continuous? Or can it have a finite number of points of discontinuity?" For the "specialist," this formulation will appear—very normally—"naïve"; however, it is by starting from this formulation that the inquiry will have to be performed.

4.10 What assessment should be made of the acquired education?

4.10.1 Procognition and further learning

An essential component of the problem raised here is: What is the *evaluating instance* $\hat{v}$? An essential question is obviously: What is the assessed "entity"? Let us start with a "simple" case: in a class $c = [X, Y]$, a team of students X_q studies, under the supervision of a team of teachers Y_q, a certain question q. Regardless of its length, their inquiry report, validated by the class, constitutes a chapter in the class book of c. It is then this class book that is being evaluated by an evaluating instance $\hat{v}$, which can be the set Y of teachers in the class or a part of that set. Of course, and here we return to a question already discussed, $y \in Y$ must be trained in this type of evaluation, which is generally codisciplinary in nature, even if it seems that our spontaneous belief in our capacity to evaluate *any* situation is an adaptive ability of our species... But the instance $\hat{v}$ can also be a body of inspectors of schools and classes. We touch here, then, on an essential difficulty, which arises at the level of society (and therefore of civilization): the recognition of the value of education acquired by questioning the world, in opposition to education acquired by visiting works.

How should this education be evaluated? The traditional answer is: by evaluating what the persons who have received this education *know*, that is to say their praxeological acquisitions, in terms of *praxis* and *logos*. In short, it is a question of evaluating what the person x has learned (and has not yet unlearned). In the paradigm of questioning the world, the criterion is different: What matters is not exactly what x *has learned* and what x will be able to do with what he has learned, but what he will be *able to learn thereafter*. Of course, what x is able to learn depends partly on what he has previously learned, but only partly, because many praxeologies (i.e., "bodies of knowledge") that have been "acquired" are or become, de facto, *unproductive*.

Any evaluation involves an instance $\hat{\imath}$, that is to say a person x or an institutional position (I, p), and a system of works $\mathcal{W}$. What is being evaluated is the relation $R(\hat{\imath}, \mathcal{W})$. To evaluate this relation, $\hat{v}$ must analyze the power of thought (*logos*) and action (*praxis*) that $R(\hat{\imath}, \mathcal{W})$ confers on $\hat{\imath}$. This is where *two* choices are possible. We will mention them by assuming, for simplicity, that we have $\hat{\imath} = x$. The first is the *retrocognitive* choice: it consists in asking x to answer questions $\mathcal{q}$ that x is supposed to have previously studied. In this case, x has indeed inquired into $\mathcal{q}$ (in principle), even if that inquiry merely involved reading accounts of inquiries in which x would not have personally participated. The second choice is the *procognitive* choice: it consists in asking x to produce an answer $\mathcal{a}$ to a question $\mathcal{q}$ that x is not supposed to have studied—x would have to take up the inquiry anyway in view of the examination that $\hat{v}$ has to perform.

4.10.2 Mistreating questions?

Before going any further, a general remark is in order. Most people ask themselves—more or less frequently—a wide variety of questions. Note here that while any answer $\mathcal{a}$ must be "solid"—that is, must have withstood the most demanding dialectic of media and milieus possible, given the resources available—no question $\mathcal{q}$ can in principle be dismissed, on the grounds, for example, that it would be frivolous. That noted, when a person x raises a question $\mathcal{q}$, several possibilities may arise. The most common, it seems, is this: the question $\mathcal{q}$ is ignored almost as soon as it comes to x's mind and will never be considered by x—it is somehow stillborn.

The "fate" of a question $\mathcal{q}$ is not necessarily to die as soon as it is born. Frequently, x is minimally Herbartian and seeks an answer $\mathcal{a}$ by listening to or consulting easily accessible "media" (in the sense of the dialectic of media and milieus), for example close people deemed "safe" by x, or "public rumor," or a dictionary, etc. The essential problem, with devastating effects, is that many people are satisfied with an answer that is both superficial (it leaves many of the questions generated unanswered) *and* insufficiently validated (the functioning of the dialectic of media and milieus remains very limited). It is clearly this "weakly Herbartian" attitude and the obstacles it promotes that education for questioning the world must combat.

4.10.3 A simple but undeniable criterion

It is not enough, of course, for x to give up being satisfied with hearsay. We will consider a crude criterion here: the amount of time spent investigating $\mathcal{q}$. Did you study this question for 5 minutes, 50 minutes, 5 hours, 50 hours, 500 hours, 5,000 hours, 50,000 hours? In the first case, one is usually very close to an answer $\mathcal{a}$ resulting from mere hearsay. In the next two cases, we are looking at a time frame often adopted for formal exams. The fourth case, which represents about 4 months of study at a rate of 4 hours per day for example, corresponds rather to the time of investigation for a master's thesis. We leave it to the reader to imagine what might correspond to the following two durations.

Depending on the institution that wishes to assess what x can learn by asking them to investigate a question $\mathcal{q}$, the duration and, more broadly, the type of exam can likely vary greatly: a one-hour or five-hour proctored test, a take-home test to be turned in within a specified time frame (one day, for example), etc. What is most important to make an inquiry possible, or already to initiate it, are obviously the *resources* (of all kinds) available to the inquirer x. It has become rather common to provide x with a folder containing a number of printed documents (including books), which may however produce a didactization rather than an adidactization effect. All this cannot be decided without further clarification of the prevailing conditions and constraints. We would personally recommend, *as an example*, a proctored exam of, say, three hours duration, with each candidate having a connection to the Internet considered as a potentially "universal" library. Each time, the investigation of the proposed question $\mathcal{q}$ will have been carried out in advance by some y playing the role of guinea pigs in order to possibly rework the question and better specify the conditions and constraints to prevail during the test. But all this cannot be done without those involved, within society as a whole, in the transition to the paradigm of questioning the world submitting themselves to the discipline of first conducting inquiries themselves, and then trying, as far as possible, to supervise such inquiries. The mathematical reader may do this about the question raised above concerning the continuity and possible discontinuities of derived functions.

5. DISCUSSION AND CONCLUSION

The generating question that has oriented our theoretical inquiry was: "Under what conditions is a transition from the paradigm of visiting works to the paradigm of questioning the world possible, locally and globally?" The analysis suggests that such a transition is not a single event but a progressive reconfiguration of didactic systems, passing through intermediate variants of both paradigms. It is governed by transformations in three closely related dimensions: (1) the institutional positions and toposes of teachers and students; (2) the construction and functioning of the milieu, including the degree of adidacticity; and (3) the institutional relation to knowledge and ignorance that defines acceptable school activity. In this sense, the "conditions" for the transition are both *structural*, linked to institutional contracts; and *epistemological*, pertaining to the relation $R(\hat{\imath}, E)$ of persons and positions to knowledge.

A transition toward the paradigm of questioning the world becomes possible when certain local and global conditions align. Locally, movement occurs when study is genuinely organized around questions rather than predetermined works, when the milieu becomes a space that students help construct and revise, and when responsibility for inquiry is redistributed through new topogenetic arrangements. Such conditions are supported by organizational forms—SRPs, seminars, and collective work with evolving media–milieus systems—that enable students and teachers to conduct inquiries together and to legitimate provisional answers.

Globally, the transition presupposes institutional conditions that stabilize and broaden these local reorganizations: a curriculum centered on families of generating questions, multidisciplinary teacher teams, and assessment regimes that value inquiry processes. It also requires an institutional relation to knowledge that legitimizes uncertainty and procognitive engagement. However, our account of these global conditions remains necessarily speculative. While we have identified structural and epistemological requirements, we have not analyzed the political, economic, and cultural constraints that shape curriculum reform, teacher work, and assessment systems. Addressing these determinants would require a multi-level institutional inquiry that extends beyond the scope of this article. These limitations point to several directions for future research: empirical studies of didactic organizations approximating the variant P_2^{+++}, with attention to the construction and negotiation of media–milieus systems; investigations into the development of the attitudes $\hbar1$–$\hbar5$ in teacher education and professional communities; and comparative analyses of curricular and assessment reforms that either hinder or support movements towards a q-curriculum.

In conclusion, our inquiry has sought to clarify under what conditions education can move from visiting works to questioning the world. The answer we have proposed is necessarily incomplete and conjectural, but it underlines a central point: Paradigm 2 cannot be reduced to a set of methods or activities; it is a different social contract governing what it means to study. Locally, this contract can be partially enacted through SRPs, seminars, and new forms of assessment. Globally, its realization would require a profound redefinition of curriculums, teacher work, and the institutional relation to knowledge and ignorance. If education is to equip future citizens to address the pressing questions that weigh on humanity, then building the conditions for such a contract is not merely a didactic option but an institutional responsibility. The paradigm of questioning the world offers a coherent, though demanding, framework for theorizing this responsibility and for orienting both didactic research and educational policy.

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